Cumulus Parameterization & related issues

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What is the difference between Numerical Weather Prediction and Climate prediction?

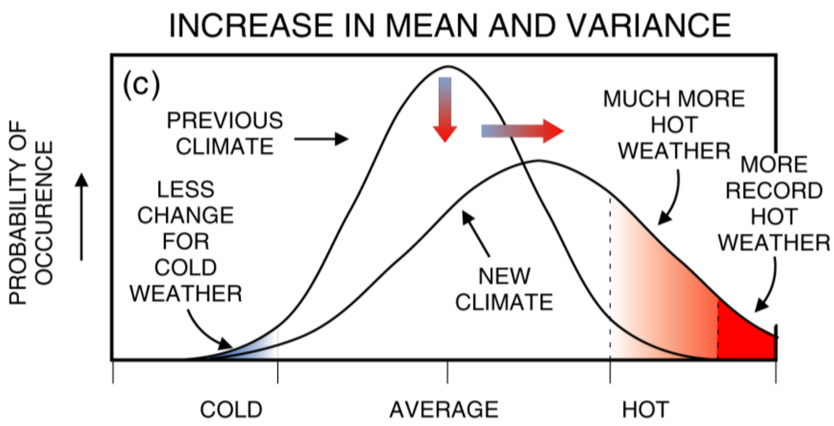
Climate v. Numerical Weather Prediction

- NWP:
  - Initial state is CRITICAL
  - Don’t really care about whole PDF, just probable phase space
  - conservation of mass/energy to match observed state

- Climate
  - Get rid of any dependence on initial state
  - Conservation of mass & energy critical
  - Want to know the PDF of all possible states
  - Don’t really care where we are on the PDF
  - Really want to know tails (extreme events)

How can we predict Climate (50 yrs) if we can't predict Weather (10 days)?

Statistics!



Conceptual Framework for Modeling

- Can’t resolve all scales, so have to represent them

- Energy Balance / Reduced Models
  - Mean State of the System
  - Energy Budget, conservation, Radiative transfer

- Dynamical Models
  - Finite element representation of system
  - Fluid Dynamics on a rotating sphere
  - Basic equations of motion
  - Advection of mass, trace species
  - Physical Parameterizations for moving energy

- Scales: Cloud Resolving/Mesoscale/Regional/Global
  - Global= General Circulation Models (GCM’s)

Physical processes regulating climate

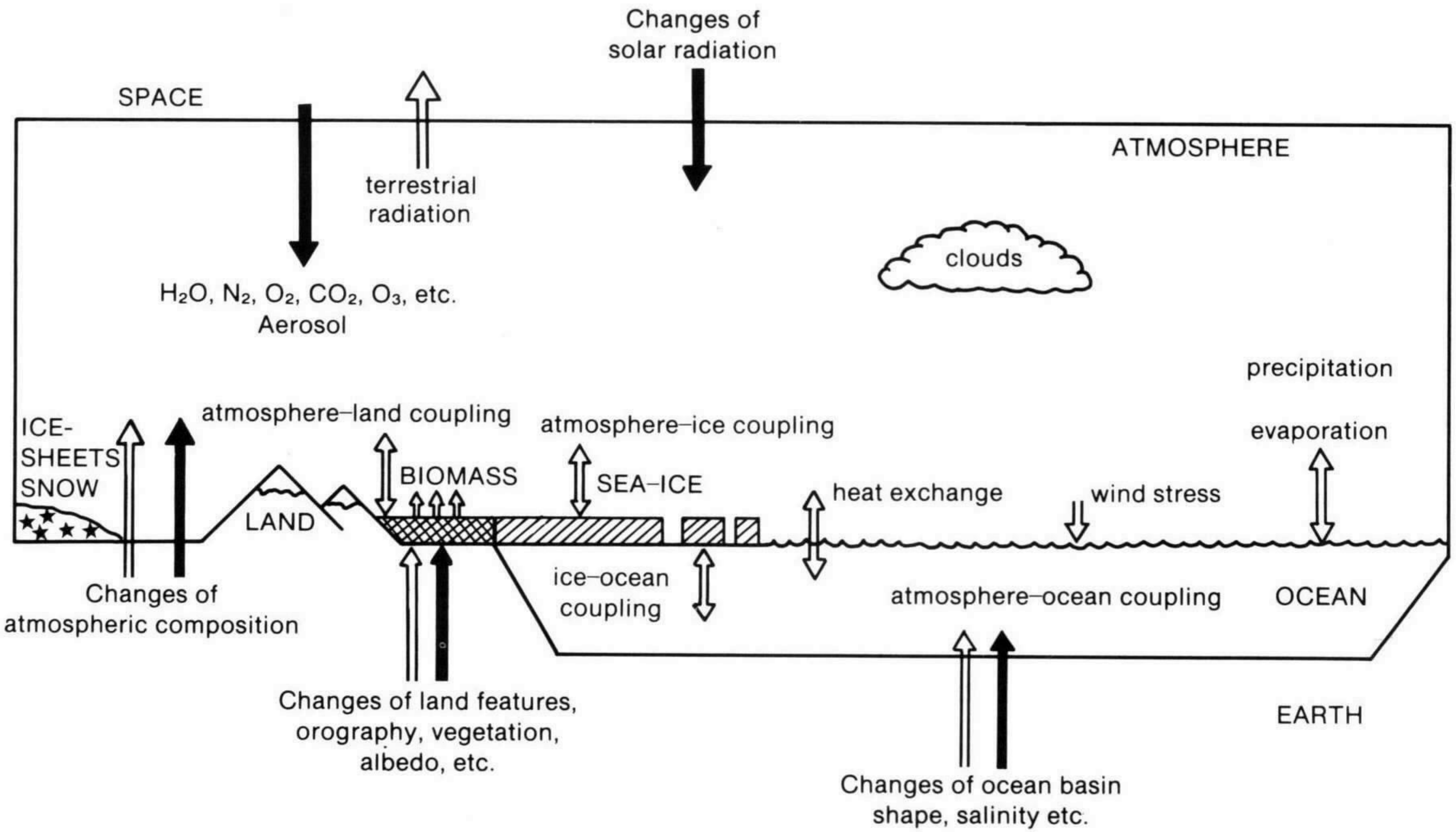
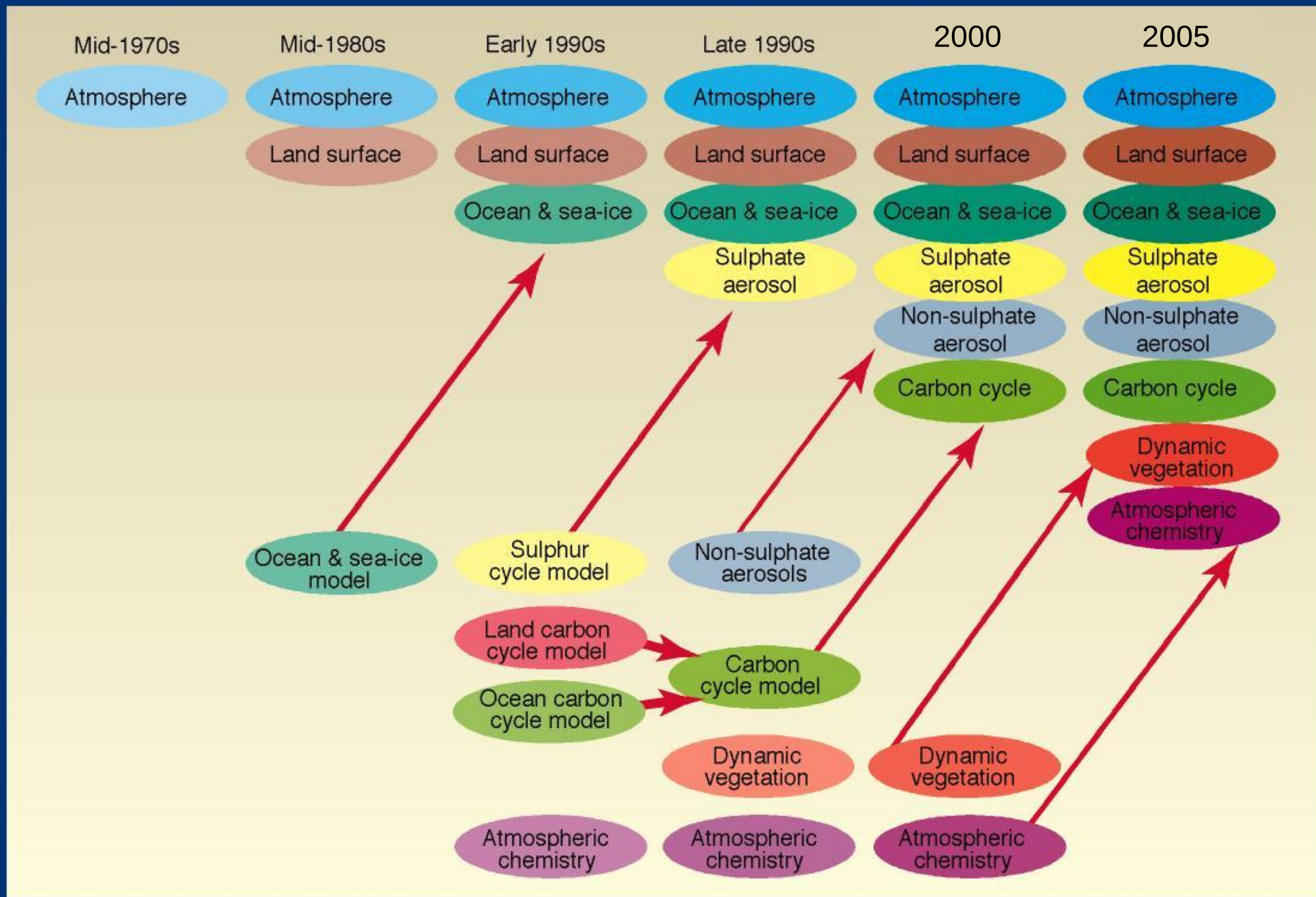


Figure 3.1: Schematic illustration of the components of the coupled atmosphere-ocean-ice-land climatic system. The full arrows are examples of external processes, and the open arrows are examples of internal processes in climatic change (from Houghton, 1984).

Earth System Model 'Evolution'

The development of climate models, past, present and future



WG1 - TS BOX 3
FIGURE 1

“Primitive” Equations

- 3 Equations of Motion: Newton’s Second Law
- First Law of Thermodynamics
- Conservation of mass
- Perfect Gas Law
- Conservation of water

With sufficient data for initialization and a mean to integrate these equations, numerical weather prediction is possible.

Example: Newton’s Second Law: F = ma

One Form

- the geostrophic momentum equations

$$\frac{Du}{Dt} - fv = -\frac{\partial \phi}{\partial x}$$
$$\frac{Dv}{Dt} + fu = -\frac{\partial \phi}{\partial y}$$

- the hydrostatic equation, a special case of the vertical momentum equation in which there is no background vertical acceleration.

$$0 = -\frac{\partial \phi}{\partial p} - \frac{RT}{p}$$

- the continuity equation, connecting horizontal divergence/convergence to vertical motion

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

- and the Thermodynamic Energy equation, a consequence of the first law of thermodynamics

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \omega\left(\frac{\partial T}{\partial p} + \frac{RT}{pc_p}\right) = \frac{J}{c_p}$$

Meteorological Primitive Equations

- Applicable to wide scale of motions; > 1hour, >100km

$$d\overline{\mathbf{V}}/dt + fk \times \overline{\mathbf{V}} + \nabla\overline{\phi} = \mathbf{F}, \qquad \textit{(horizontal momentum)}$$

$$d\overline{T}/dt - \kappa\overline{T}\omega/p = Q/c_p, \qquad \textit{(thermodynamic energy)}$$

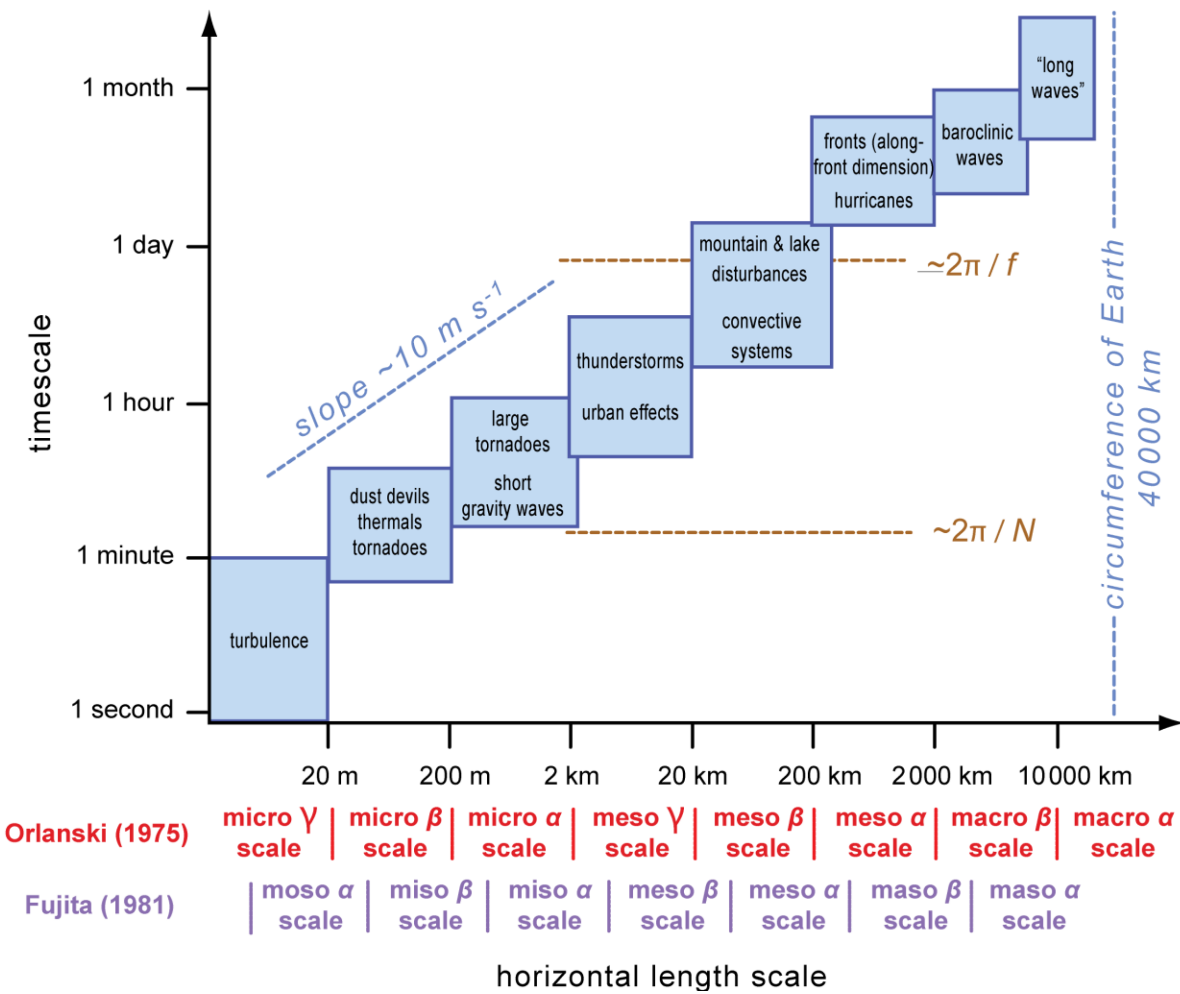
$$\nabla \cdot \overline{\mathbf{V}} + \partial\overline{\omega}/\partial p = 0, \qquad \textit{(mass continuity)}$$

$$\partial\overline{\phi}/\partial p + R\overline{T}/p = 0, \qquad \textit{(hydrostatic equilibrium)}$$

$$d\overline{q}/dt = S_q. \qquad \textit{(water vapor mass continuity)}$$

Harmless looking terms $\mathbf{F}$, Q, and $S_q \Longrightarrow$ "physics"

Scales of atmospheric motion



- *Atmospheric motions occur over a broad continuum of space and time scales. The mean free path of molecules (approximately 0.1 μm) and circumference of the Earth place lower and upper bounds on the space scales of motions.*

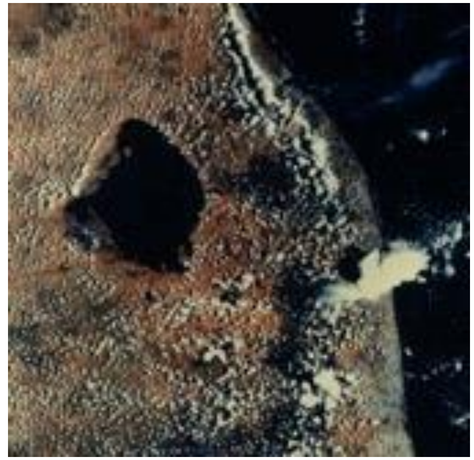
- *The timescales of atmospheric motions range from under a second, in the case of small-scale turbulent motions, to as long as weeks in the case of planetary-scale Rossby waves.*

- *Meteorological phenomena having short temporal scales tend to have small spatial scales, and vice versa; the ratio of horizontal space to time scales is of roughly the same order of magnitude for most phenomena (~10 $m\ s^{-1}$)*

Scale analysis for three specific phenomena

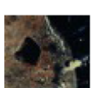
SYNOPTIC SCALE
extra-tropical cyclone

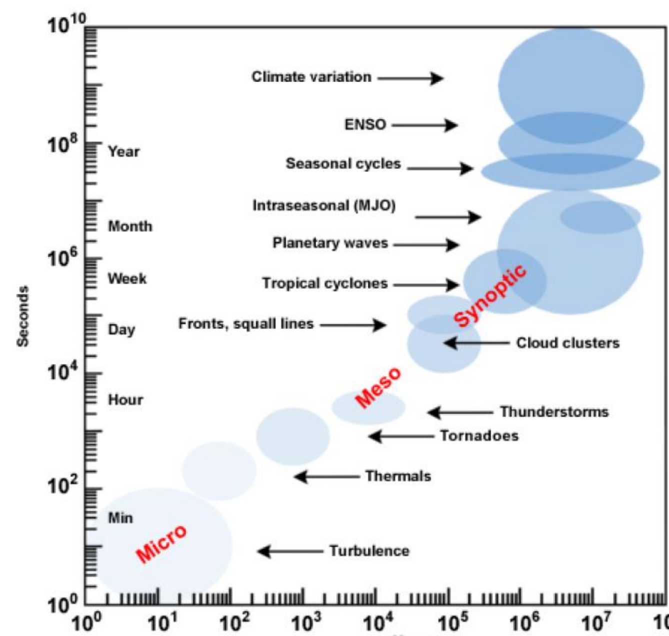
MESOSCALE
Sea-breeze



MICROSCALE
Cumulus cloud

Some characteristic values

System (Scale)	Time	Length Scales		**Velocity fields**		**Thermodynamic fields**	
	[T] s	Horizontal [L] m	Vertical [D] m	Horizontal [u] ms^{-1}	Vertical [L] ms^{-1}	Pressure [L] m^2s^{-2}	Pot. Temp. [L] K
 Depression (Macro)	$> 10^5$	10^6	10^4	10	$(<10^{-2})$	10^3	4
 Sea breeze (Meso)	10^4	5×10^4	2×10^3	3	$(<10^{-1})$	10^2	2
 Cumulus (Micro)	5×10^2	2×10^3	5×10^3	1	3	10	10^{-1}



Scale Analysis for synoptic-scale motions

***Scale analysis, or scaling*, is a convenient technique for estimating the magnitudes of various terms in the governing equations for a particular type of motion.**

Element	Typical value	Magnitude
U,V (horizontal velocity)	**10 m s^{-1}**	10^1
W (vertical velocity)	1 cm s^{-1}	10^{-2}
L (length, distance scale)	**1000 km**	10^6
H (depth, height scale)	10 km (depth of troposphere)	10^4
Horizontal pressure gradient	10 hPa	10^3
Vertical pressure gradient	1000 hPa	10^5
Time (L/U)	**of the order of 1 day**	10^5
ρ (density)	1 kg m^{-3}	10^0
g (gravity)	9.8 m s^{-2}	10^1
Ω (angular velocity)	7.292×10^{-5} s^{-1}	10^{-4}

Synoptic scale motions- Scale analysis

Horizontal momentum equations

$$\frac{du}{dt} - 2\Omega v \sin\varphi + 2\Omega w \cos\varphi = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{dv}{dt} + 2\Omega u \sin\varphi = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

U^2/T	$\lvert 2\Omega U \sin\varphi \rvert$	$\lvert 2\Omega W \cos\varphi \rvert$	$\frac{\Delta p_h}{\rho L}$	(scales)
10^{-4}	10^{-3}	10^{-6}	10^{-3}	(orders)

Vertical momentum equation

$$\frac{dw}{dt} - 2\Omega u \cos\varphi = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$\frac{UW}{L}$	$\lvert 2\Omega u \cos\varphi \rvert$	$\frac{\Delta p_v}{\rho H}$	g	(scales)
10^{-7}	10^{-3}	10	10	(orders)

Synoptic scale motions :

$L = 1000 \text{ km} = 10^6 \text{ m}$

$T = 1 \text{ day } (10^5 \text{ sec})$

$U = 10 \text{ m s}^{-1}$

$H = 10 \text{ km}$

$\Delta p_{horizontal} = 10 \text{ hPa}$

$\Delta p_{vertical} = 1000 \text{ hPa}$

$\rho = 1 \text{ kg m}^{-3}$

$f \text{ (at } 45^0 \text{ N)} \sim 10^{-4} \text{ s}^{-1}$

Bottomline :

The synoptic scale motions tend to approach geostrophic balance and in hydrostatic balance

Global Climate Model Physics

Terms *F*, *Q*, and S_q represent physical processes

- Equations of motion, *F*
  - turbulent transport, generation, and dissipation of momentum
- Thermodynamic energy equation, *Q*
  - convective-scale transport of heat
  - convective-scale sources/sinks of heat (phase change)
  - radiative sources/sinks of heat
- Water vapor mass continuity equation
  - convective-scale transport of water substance
  - convective-scale water sources/sinks (phase change)

Grid Discretizations

Equations are distributed on a sphere

- Different grid approaches:
  - Rectilinear (lat-lon)
  - Reduced grids
  - ‘equal area grids’: icosahedral, cubed sphere
  - Spectral transforms
- Different numerical methods for solution:
  - Spectral Transforms
  - Finite element
  - Lagrangian (semi-lagrangian)
- Vertical Discretization
  - Terrain following (sigma)
  - Pressure
  - Isentropic
  - Hybrid Sigma-pressure (most common)

Model Physical Parameterizations

Physical processes breakdown:

- Moist Processes
  - Moist convection, shallow convection, large scale condensation
- Radiation and Clouds
  - Cloud parameterization, radiation
- Surface Fluxes
  - Fluxes from land, ocean and sea ice (from data or models)
- Turbulent mixing
  - Planetary boundary layer parameterization, vertical diffusion, gravity wave drag

Basic Logic in a GCM (Time-step Loop)

For a grid of atmospheric columns:

1. 'Dynamics': Iterate Basic Equations
   Horizontal momentum, Thermodynamic energy,
   Mass conservation, Hydrostatic equilibrium,
   Water vapor mass conservation
2. Transport 'constituents' (water vapor, aerosol, etc)
3. Calculate forcing terms ("Physics") for each column
   Clouds & Precipitation, Radiation, etc
4. Update dynamics fields with physics forcings
5. Gravity Waves, Diffusion (fastest last)
6. Next time step (repeat)

Physics Parameterizations

- We need physics parameterizations to include key physical processes.
- Examples include radiation, cumulus convection, cloud microphysics, boundary layer physics, etc.
- Why? Primitive equations with lack the necessary physics or lack sufficient resolution to resolve key processes.

Parameterization

- Example: Cumulus Parameterization
- Most numerical models (grid spacing of 12-km is the best available operationally) cannot resolve convection (scales of a few km or less).
- In parameterization, represent the effects of sub-grid scale cumulus on the larger scales.

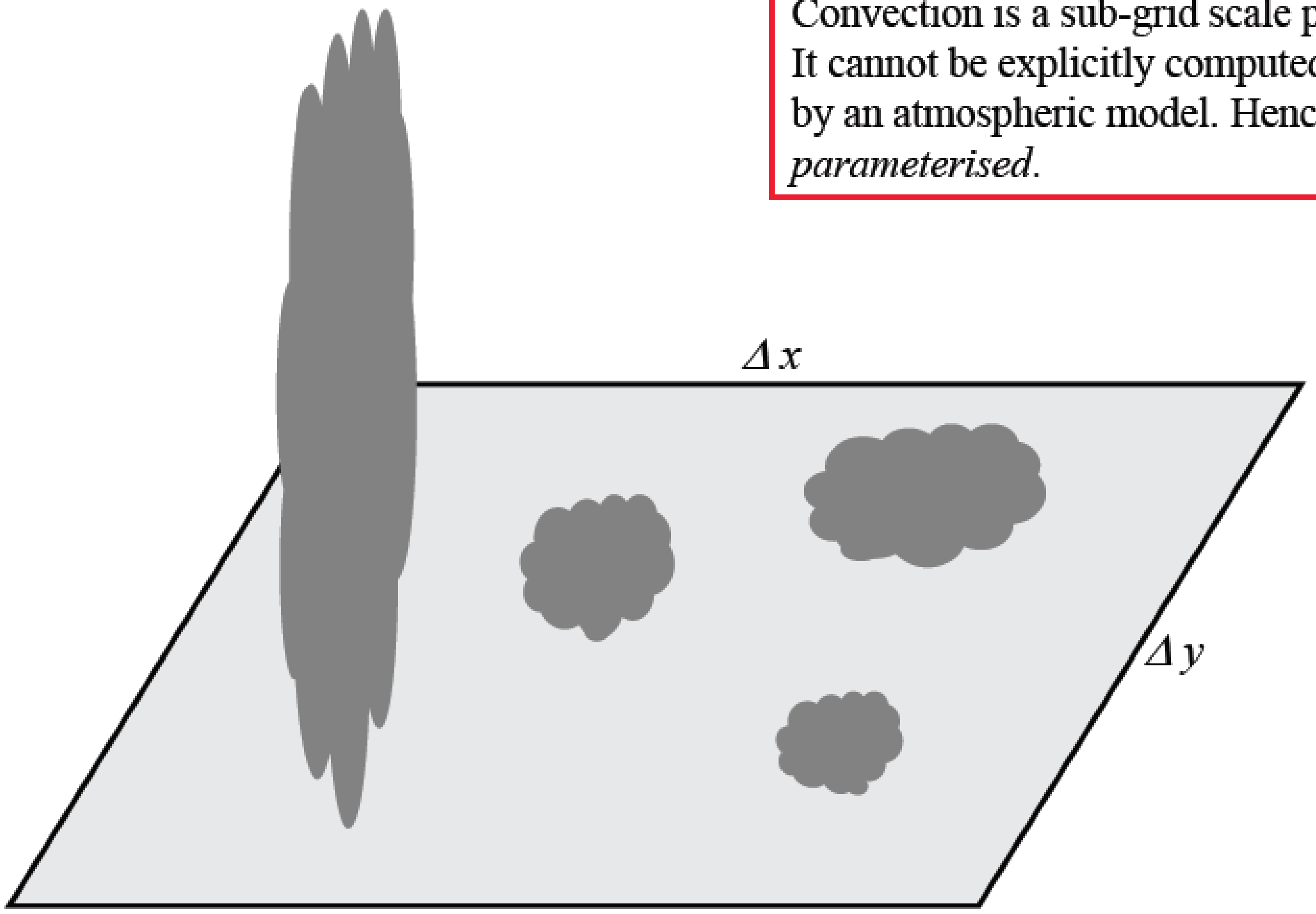
Physical Parameterization

To close the governing equations, it is necessary to incorporate the effects of physical processes that occur on scales below the numerical truncation limit

- Physical parameterization
  - express unresolved physical processes in terms of resolved processes
  - generally empirical techniques
- Examples of parameterized physics
  - dry and moist convection
  - cloud amount/cloud optical properties
  - radiative transfer
  - planetary boundary layer transports
  - surface energy exchanges
  - horizontal and vertical dissipation processes
  - ...

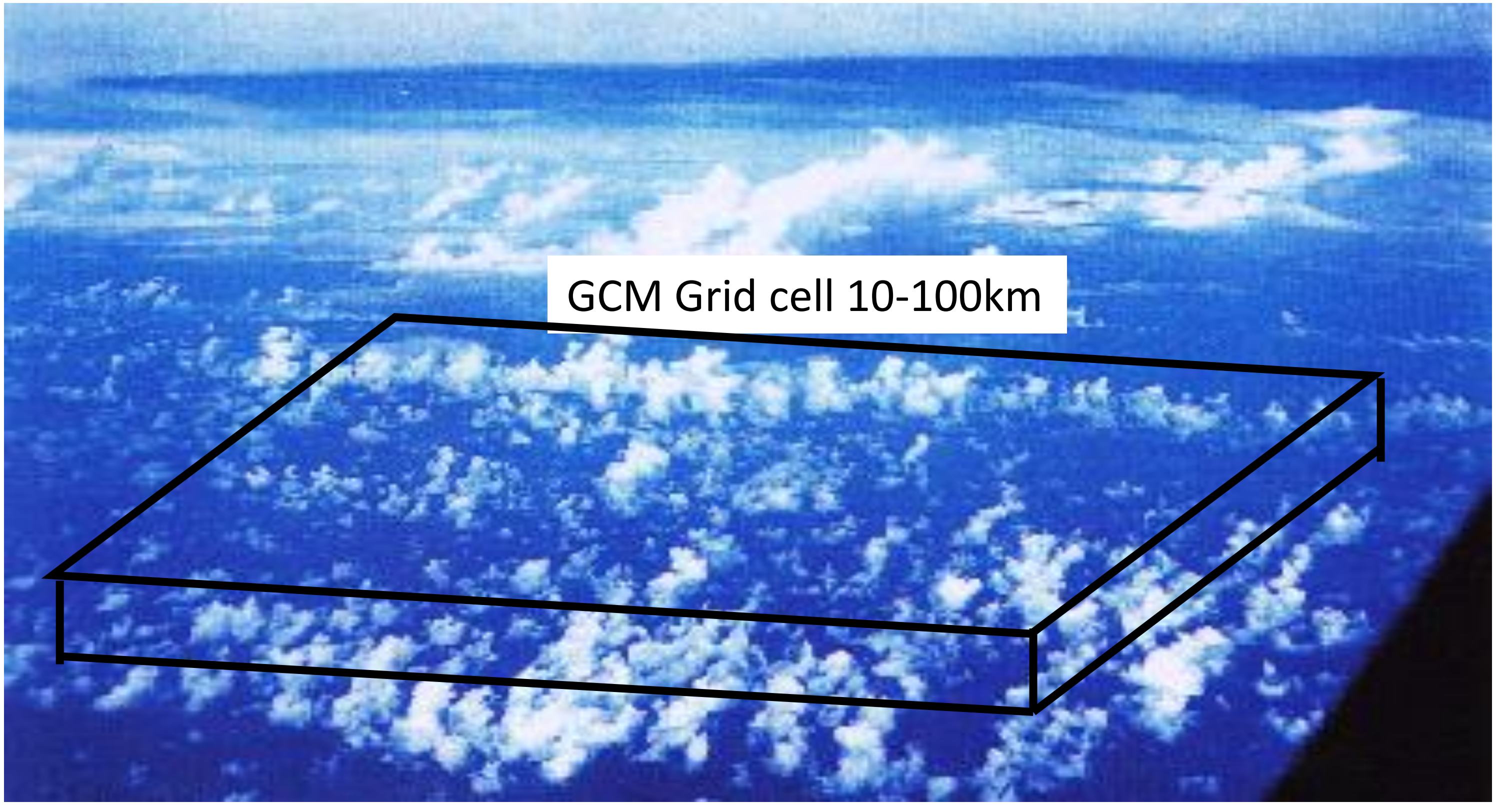
The Need for a Parameterisation

Convection is a sub-grid scale phenomenon. It cannot be explicitly computed (resolved) by an atmospheric model. Hence, it should be *parameterised*.



Clouds in GCMs - What are the problems ?

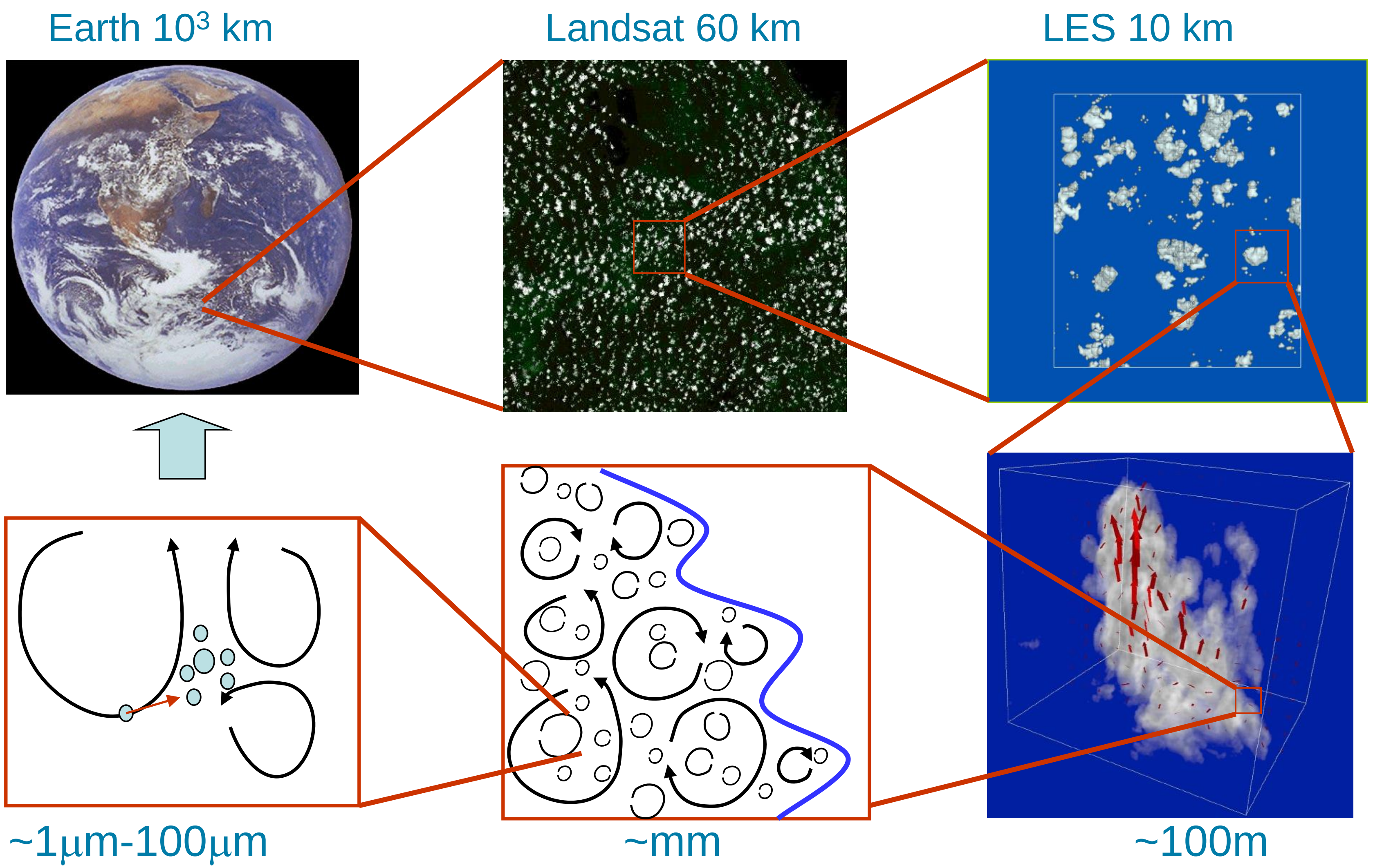
Many of the observed clouds and especially the processes within them are of subgrid-scale size (both horizontally and vertically)



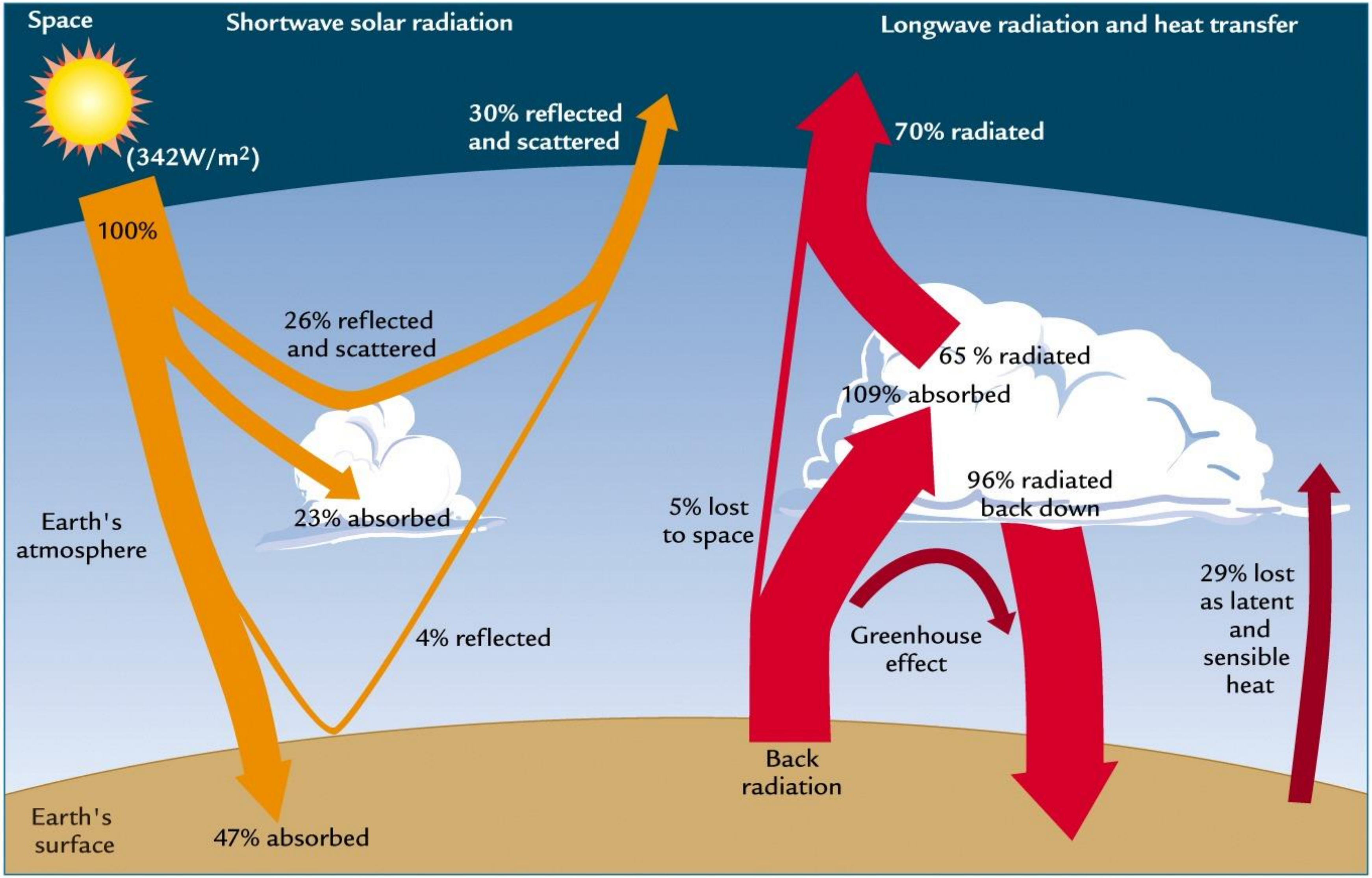
Flow chart of lecture on Convective parameterization

- What comes to the mind when we talk of moist convection?
- Why is it important and what are the different types of moist convection?
- Moist process-A multi-scale problem
- What is convective parameterization and why is it necessary?
- Point of uncertainties in convective parameterization
- Few well known schemes: KUO scheme, Arakawa-Schubert, Betts-Miller-Janjic and Kain-Fritsch

Length scales in the atmosphere

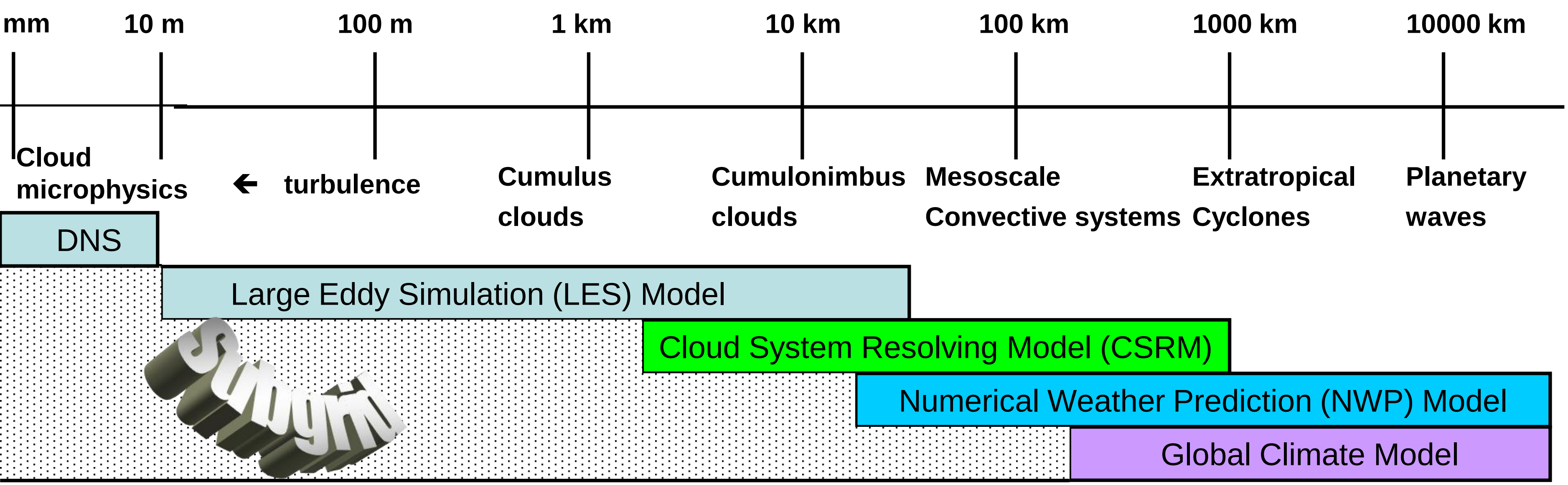


Global mean turbulent heat fluxes



source: Ruddiman, 2000

No single model can encompass all relevant processes

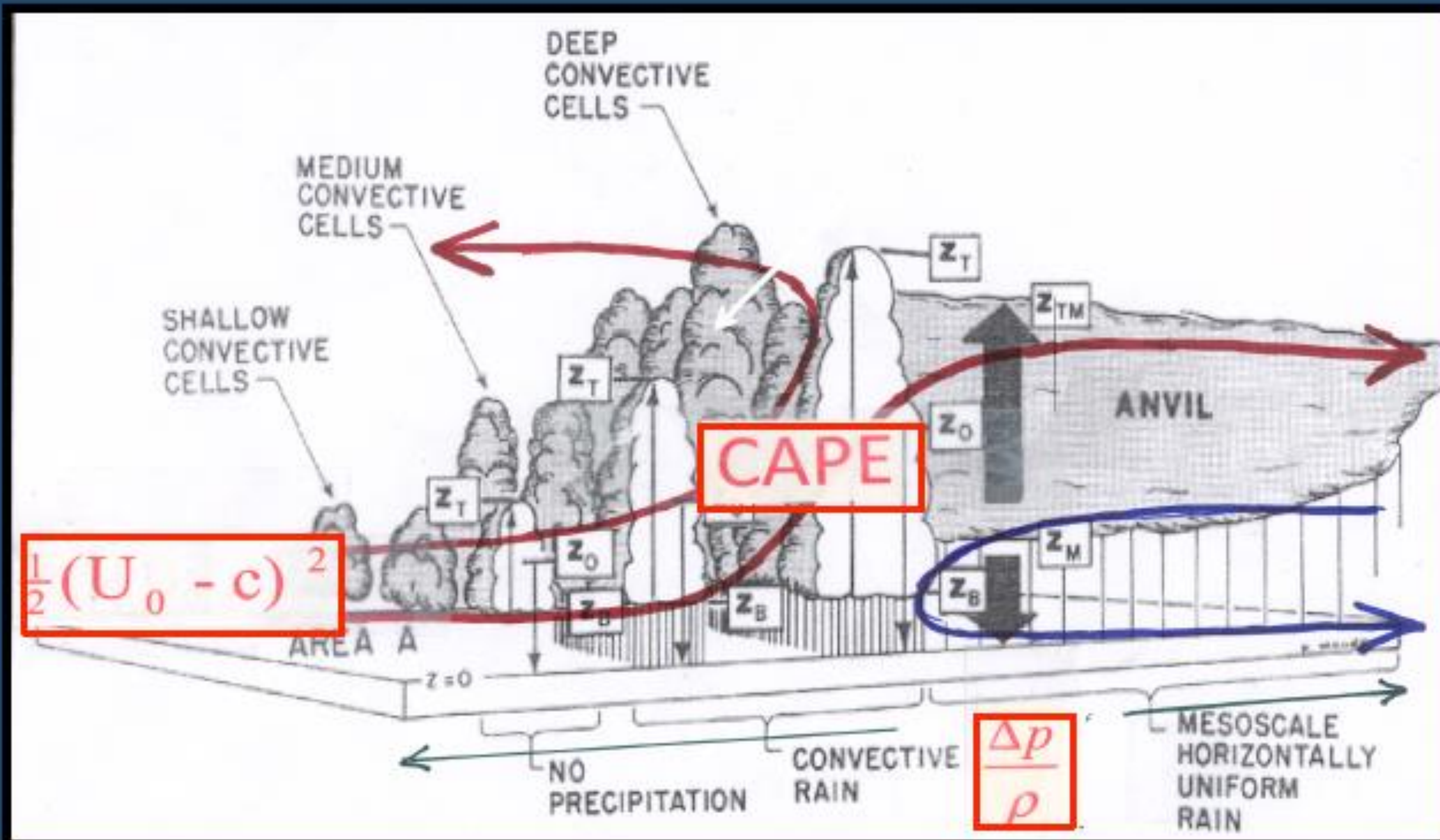


Stratiform convection

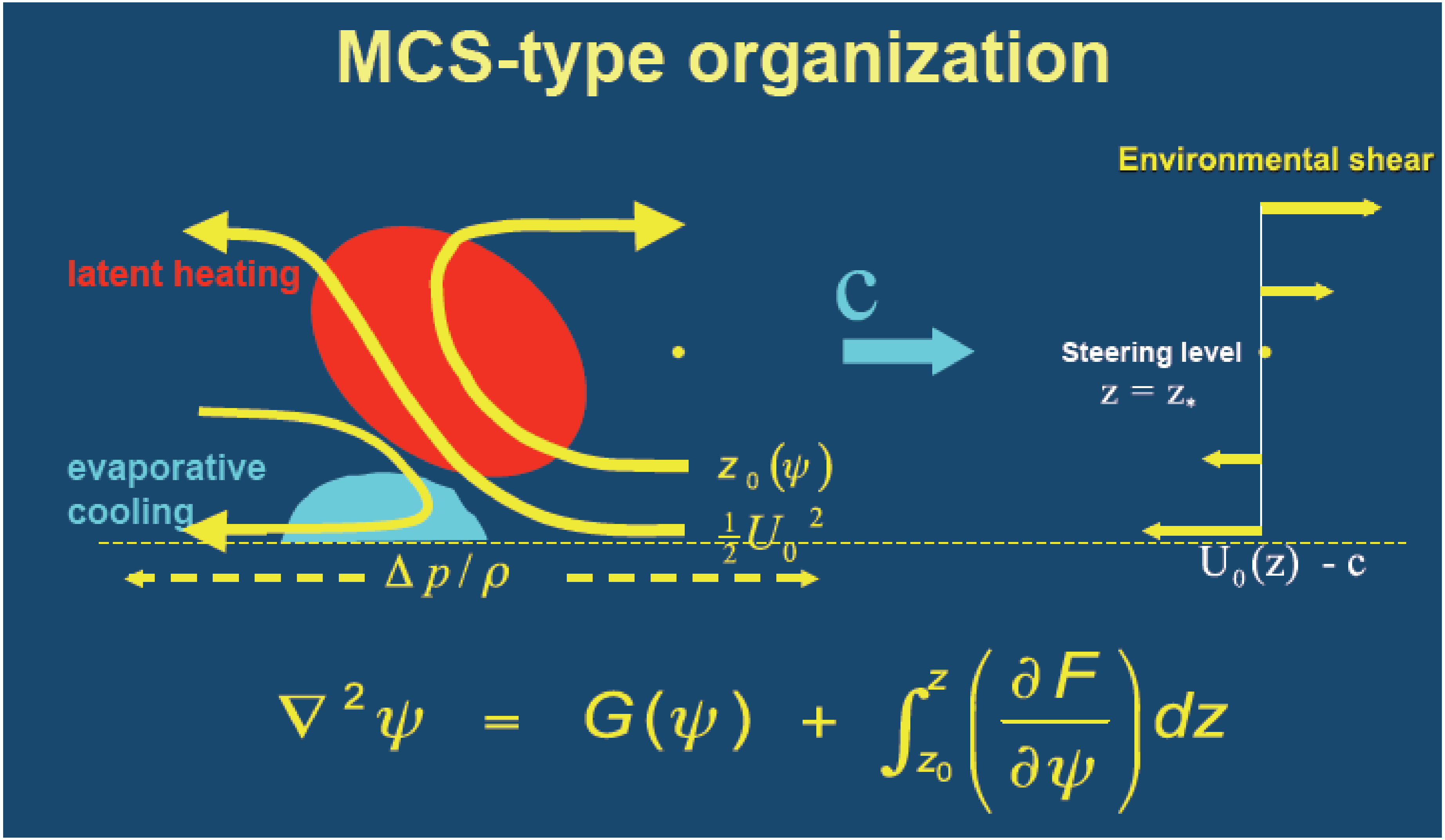
Deep convection can be further sub divided into convective and stratiform components (Houze, 1997, Chattopadhyay etal, 2009). The convective components refer to convection associated with individual cells, horizontally small regions of more intense updrafts and down drafts in association with young and active convection.

The stratiform component refers to convection associated with older, less active convection with vertical motion generally less than $1ms^{-1}$.

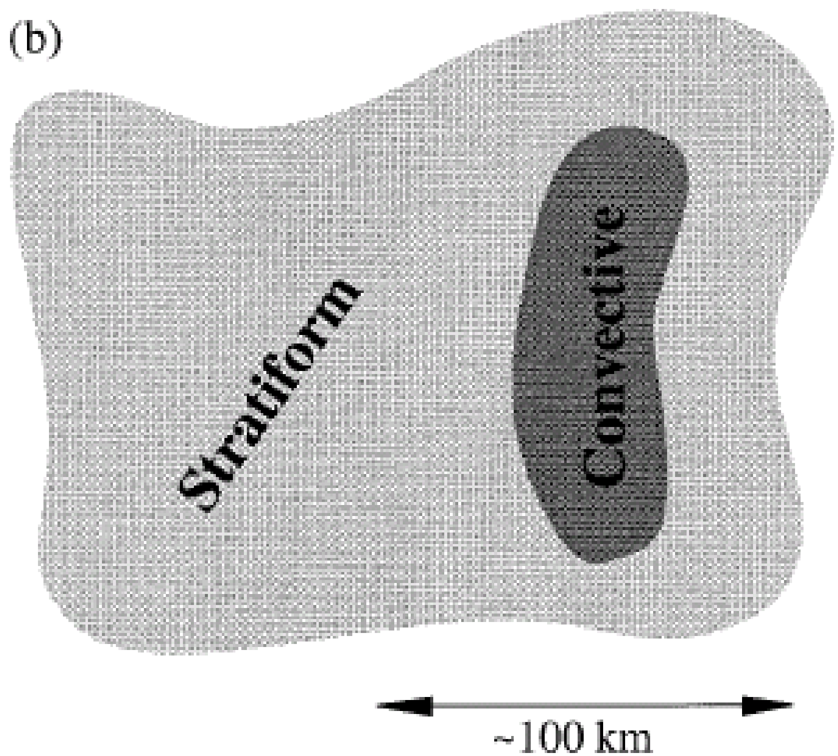
Energetics of MCS-type organization



Adopted from Bob Houze



Adopted from Mitch Moncrieff



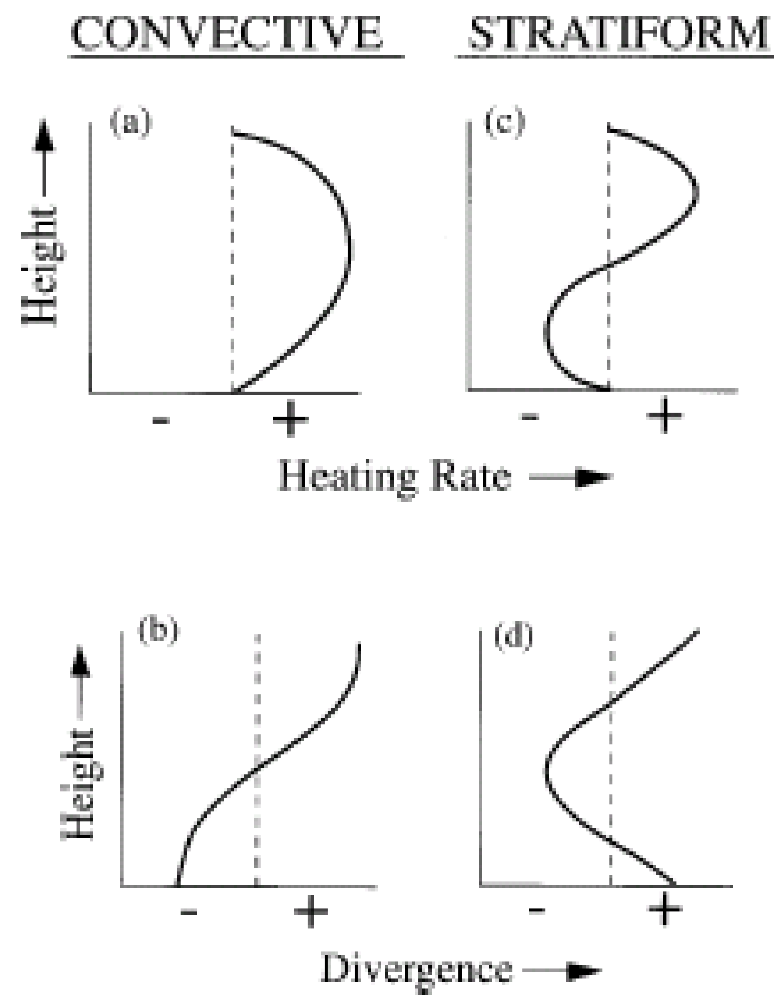


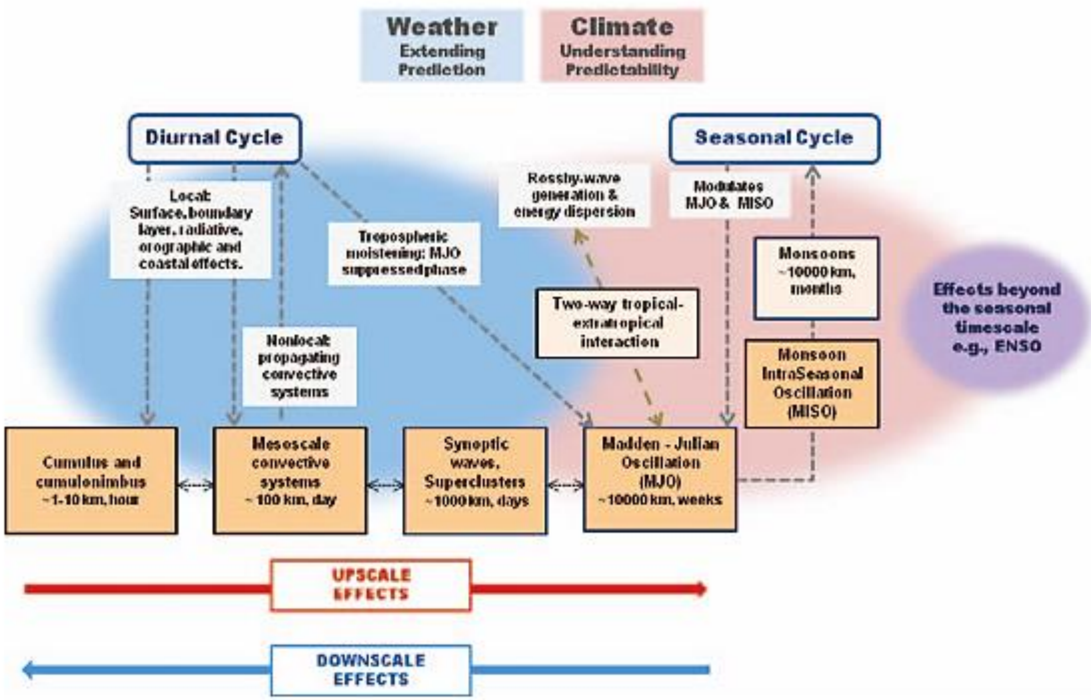
FIG. 3. Characteristic profiles of latent heating and horizontal mass divergence in convective and stratiform regions of tropical precipitation.

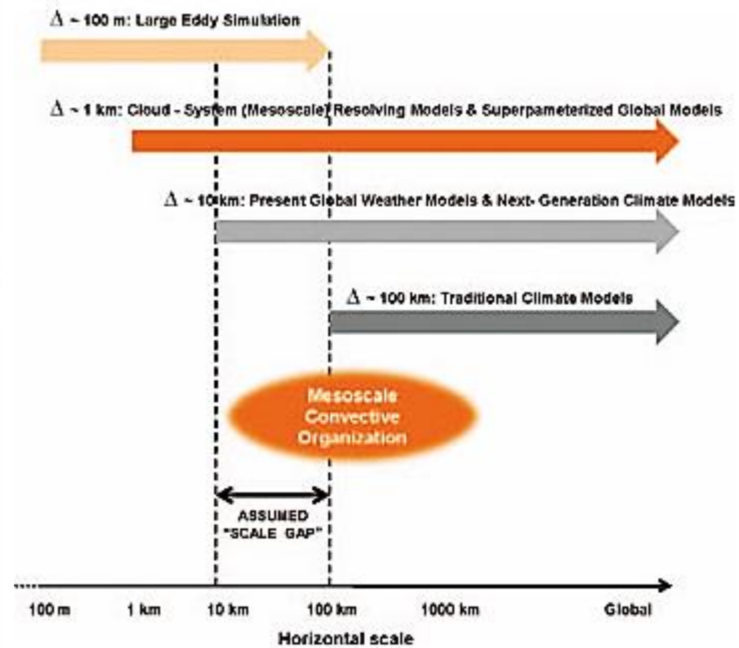
Multi-scale nature

- Essentially moist convection is comprised of two components namely convective and stratiform which has different spatio-temporal scale. This is the reason why convection is a multi-scale process.
- The present day challenge is to devise a scheme (parameterization) that can resolve the multi-scale nature of convection in a realistic way.

Moncrieff et al, 2012, BAMS

Scientific Basis of the study





The organized systems exhibit hierarchical coherence: (i) mesoscale systems consist of families of cumulonimbus; (ii) cumulonimbus and MCS are embedded in synoptic waves; and (iii) the MJO/MISO is an envelope of cumulonimbus, MCS, and superclusters.

The upscale effects of convective organization are not represented in traditional climate models.

The mean atmospheric state exerts a strong downscale control on convective structure, frequency, and variability. Mesoscale convective organization bridges the scale gap assumed in traditional convective parameterization.

(i) SCM/CRM resolves cumulus, cumulonimbus, mesoscale circulations, but the computational domain is small (~100 km) and simulations short (~1 day).

(ii) Two-dimensional CSRMs in superparameterized global models permit MCS-type organization and mesoscale dynamics.

(iii) High-resolution global numerical prediction models may crudely represent large MCS (superclusters). (iv) MCS, and other mesoscale dynamical systems, are absent from traditional climate models—organized convection is not parameterized.

Issues identified as Grand challenge by WCRP: on Cloud and convection processes are as follows

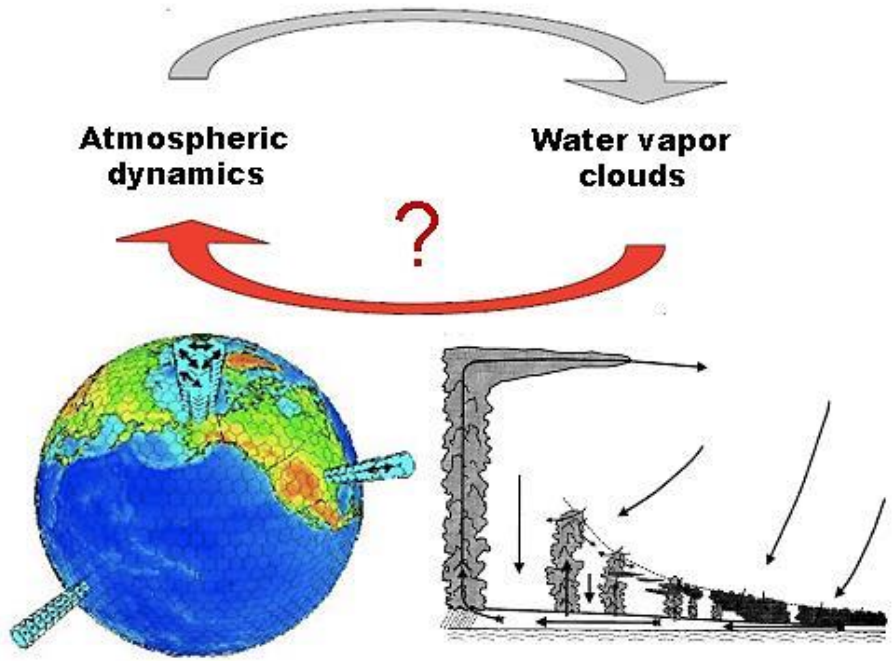
WCRP Grand Challenge on Clouds, Circulation and Climate Sensitivity

White Paper on WCRP Grand Challenge #4 Sandrine Bony and Bjorn Stevens, Nov, 2012

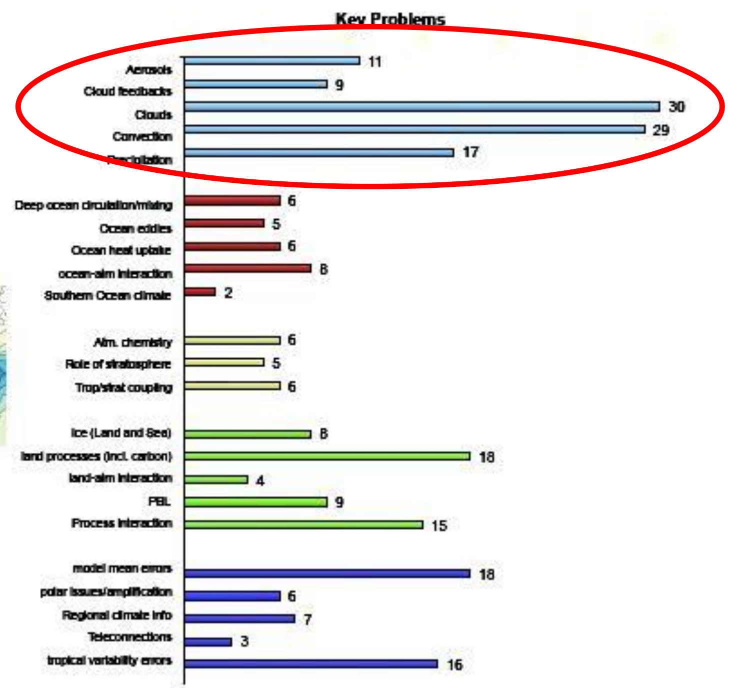
Limited understanding of clouds is the major source of uncertainty in climate sensitivity, but it also contributes substantially to persistent biases in modelled circulation systems.
As one of the main modulators of heating in the atmosphere, clouds control many other aspects of the climate system

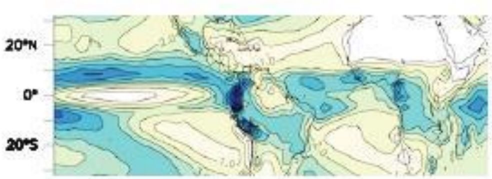
Initiative on coupling clouds to circulation (Dr. Siebesma and Frierson)

Tackle the parameterization problem through a better understanding of the interaction between cloud / convective processes and circulation system
Lessons from observations and cloud-resolving modelling over large domains; Interaction between diabatic heating and large-scale dynamics.



Source: https://www.wcrp-climate.org/gc-clouds-circulation-activities/gc4-clouds-initiatives/114-gc-clouds-inititative2





Initiative - towards more reliable models
Led by Dr. Christian Jakob (Monash Univ., Australia) & Masahiro Watanabe (Tokyo Univ., Japan)
Aim: Interpret and reduce model errors to gain confidence in projections and predictions.
Focus: Long-standing model biases (at least a few of them); Understand how model errors or shortcomings impact projections and predictions; Gain physical understanding of the climate system through model development.

What is parameterization and why is it necessary?

The basic physical equations describe the behavior of the atmosphere on small scales.
From these we derive equations that describe the behavior of the system on larger scales.

The large-scale equations contain terms that represent the effects of smaller-scale processes.
A "parameterization" is designed to represent the effects of the smaller-scale processes in terms of the large-scale state.
Since cumulus parameterization is an attempt to formulate the statistical effects of cumulus convection without predicting individual clouds, it is a closure problem in which we seek a limited number of equations that govern the statistics of a system with huge dimensions. Therefore, the core of the cumulus parameterization problem, as distinguished from the dynamics and thermodynamics of individual clouds, is in the choice of appropriate closure assumptions. (Arakawa, Met. Monograph, 1993)
Parameterizations are much more than curve fits. They are statistical theories that describe the interactions of small scales with larger scales. Parameterizations typically involve idealizations as well as "closure assumptions" that are, at best, only approximately valid.

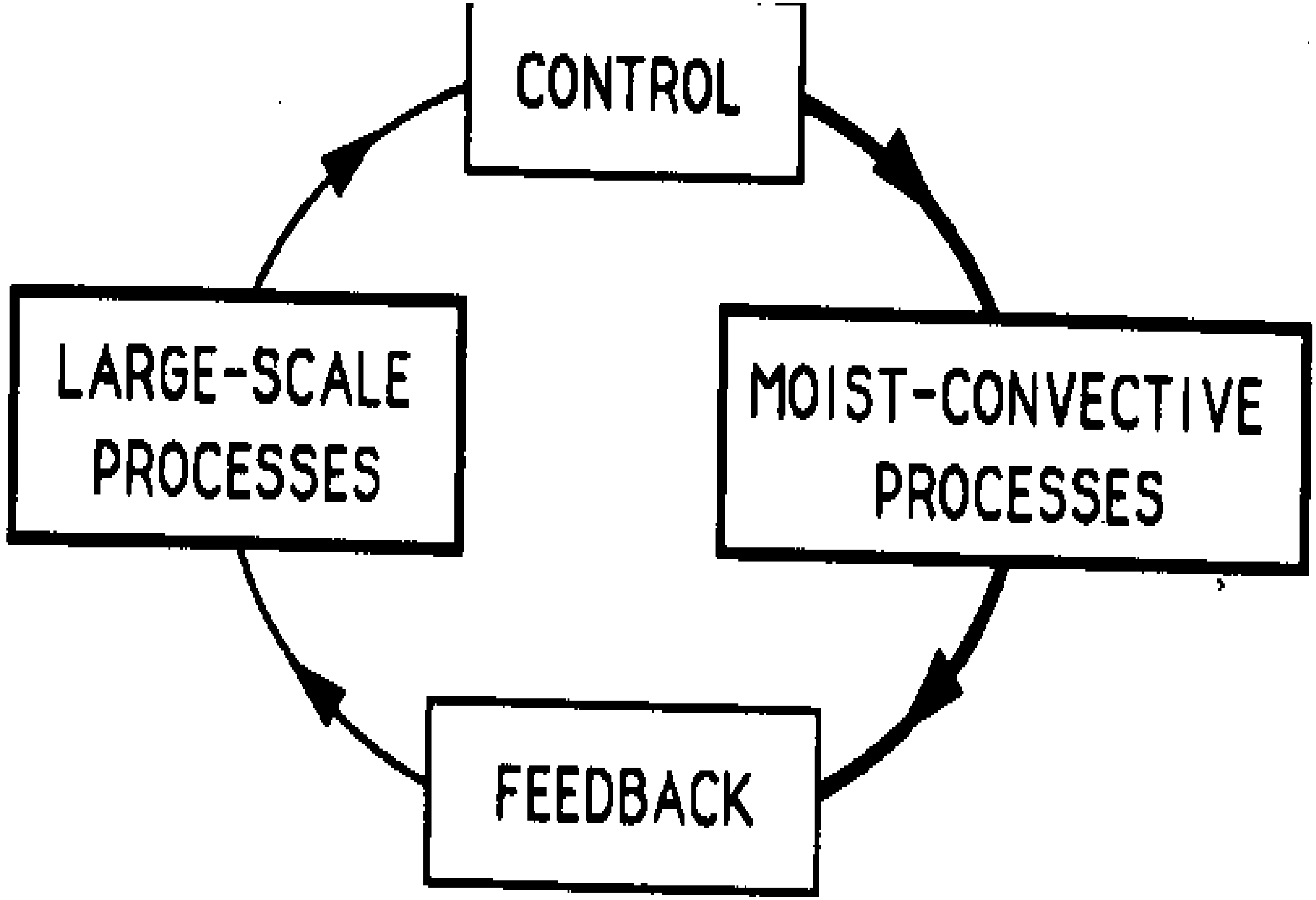


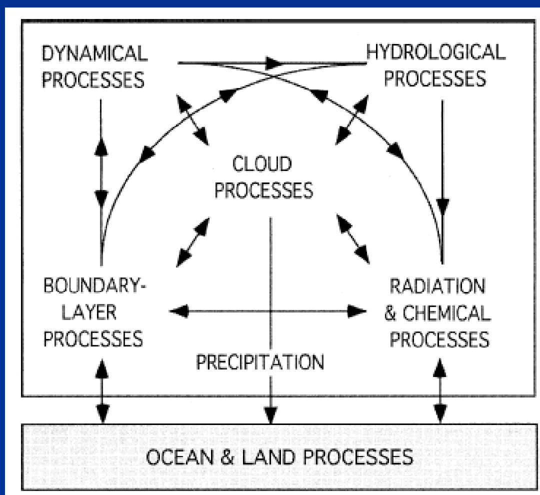
FIG. 1.1. A schematic figure showing the interaction between large-scale and moist-convective processes.

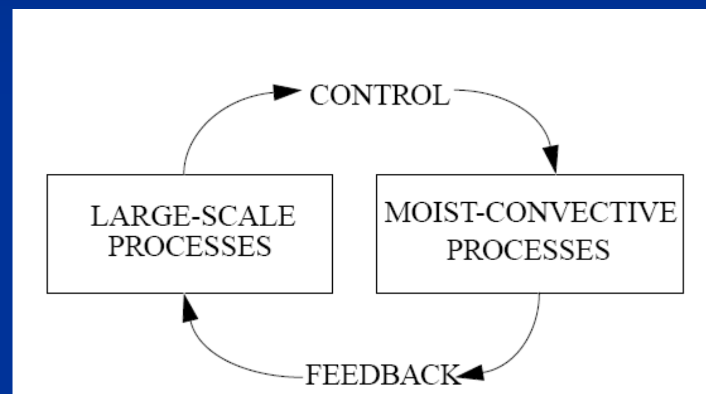
Arakawa, Met. Mono. No.46, 1993

Issues of cumulus Parameterization

The Cumulus Parameterization Problem: Past, Present, and Future
By Akio Arakawa, JOC, 2004, Arakawa et al. 2011, Arakawa and Wu 2013, Wu and Arakawa 2014

- "Major practical and conceptual problems in the conventional approach of cumulus parameterization, includes inappropriate separations of processes and scales".





$$\sum_{j=1}^{N} K_{ij} \cdot M_{Bj} + F_i = 0$$

K_{ij} = effect of cloud j on cloud i,

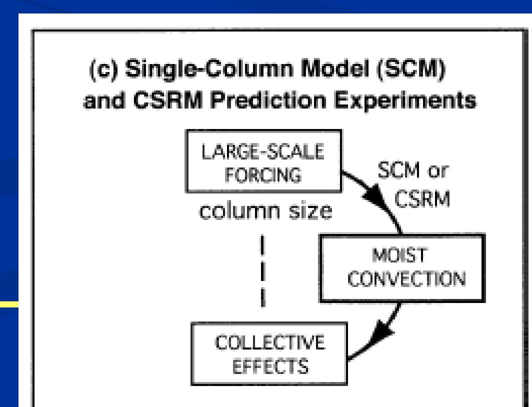
F_i = environmental forcing for cloud i

M_{Bj} = mass flux at base of cloud j

Task of Conv. Param

To calculate the collective effects of an ensemble of convective clouds in a model column

$$Q_{1C} \equiv Q_1 - Q_R \equiv L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}$$



Multi-scale clouds

Courtesy: Brian Mapes

Marine stratocumulus

Conceptualizing cumulus parameterization

- Since convective parameterization represents the effects of sub-grid scale processes on the grid variables, it is called an implicit parameterization

- Convective parameterization can be conceptualized in many ways and can be separated into some basic types (Mapes 1997).

- Convective parameterization can be grouped as deep-layer control schemes and low level control schemes.

- *Deep layer control schemes relates the creation of CAPE by large scale processes to the development of convection. These schemes could be termed "supply side" approaches as it is assumed that convection consumes the CAPE that is created.*

- *Low level control schemes tie the development of convection to the initiation processes by which CINE is removed.*

Some other way to conceptualize convection parameterization

How the environment changes due to convection?

Static schemes that determines the final environmental state after convection is done and adjusts the model fields towards this final state. It does resolve the details that produce this state. The final state is one that is neutral to convective overturning

A dynamic schemes assumes the physical processes involved in convection are important and influence the functions of the schemes. Some of these schemes use entraining plumes to approximate the effects of convection and compute the transfers of mass in updrafts and down drafts from one vertical levels to the other. (Mass flux schemes e. g. Kain and Fritsch, 1993; Tiedtke 1989)

- Convection can also be viewed as driven by buoyancy. From this instability view point local buoyancy is the key variable required to determine the convective response. Buoyancy is a key components of many convective parameterization schemes.

- Hence Buoyancy and moisture both are crucial for convective parameterization. Moisture is key component in the sense that convective parameterization is a method to account for the effects of sub-grid scale saturation. Moisture content should drive the behaviour of convective scheme by controlling amount of convection produced in an unstable environment based on available moisture that can be removed from the atmosphere.

- Closure assumptions are used to define where and when convection is activated. Closure assumptions also determine the amount and intensity of the convection and a separate set of criteria are used to determine convective development which is called "Trigger functions". Trigger functions determine how convection evolves over time.

Point of uncertainties

- There are a number of uncertainties in modeling clouds and their associated processes such as those shown below fig.

- we do not adequately understand what determines the rate of entrainment of "environmental" air into the updrafts, or how entrainment affects the evolution of a convective cloud system.

- Cumulus entrainment entails the dilution of convective updraft by dry, cool environmental air.

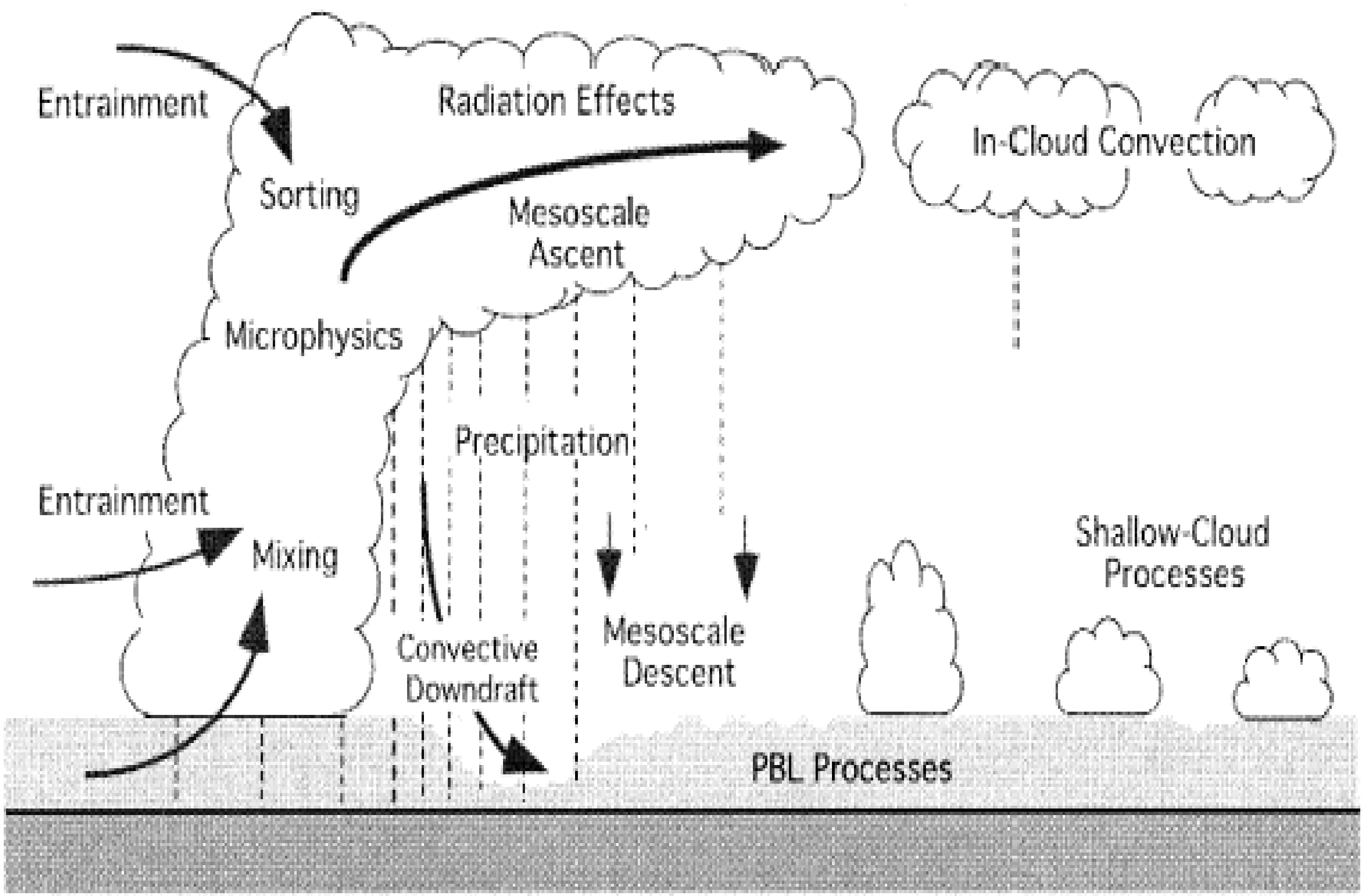
- Current parameterizations incorporate the effects of entrainment through simple assumptions (e.g., Lin and Arakawa 1997a b)

- The environment of the hot towers is typically assumed to be uniform, but in reality its properties vary on unresolved scales, due in part to the humid corpses of deceased cumuli.

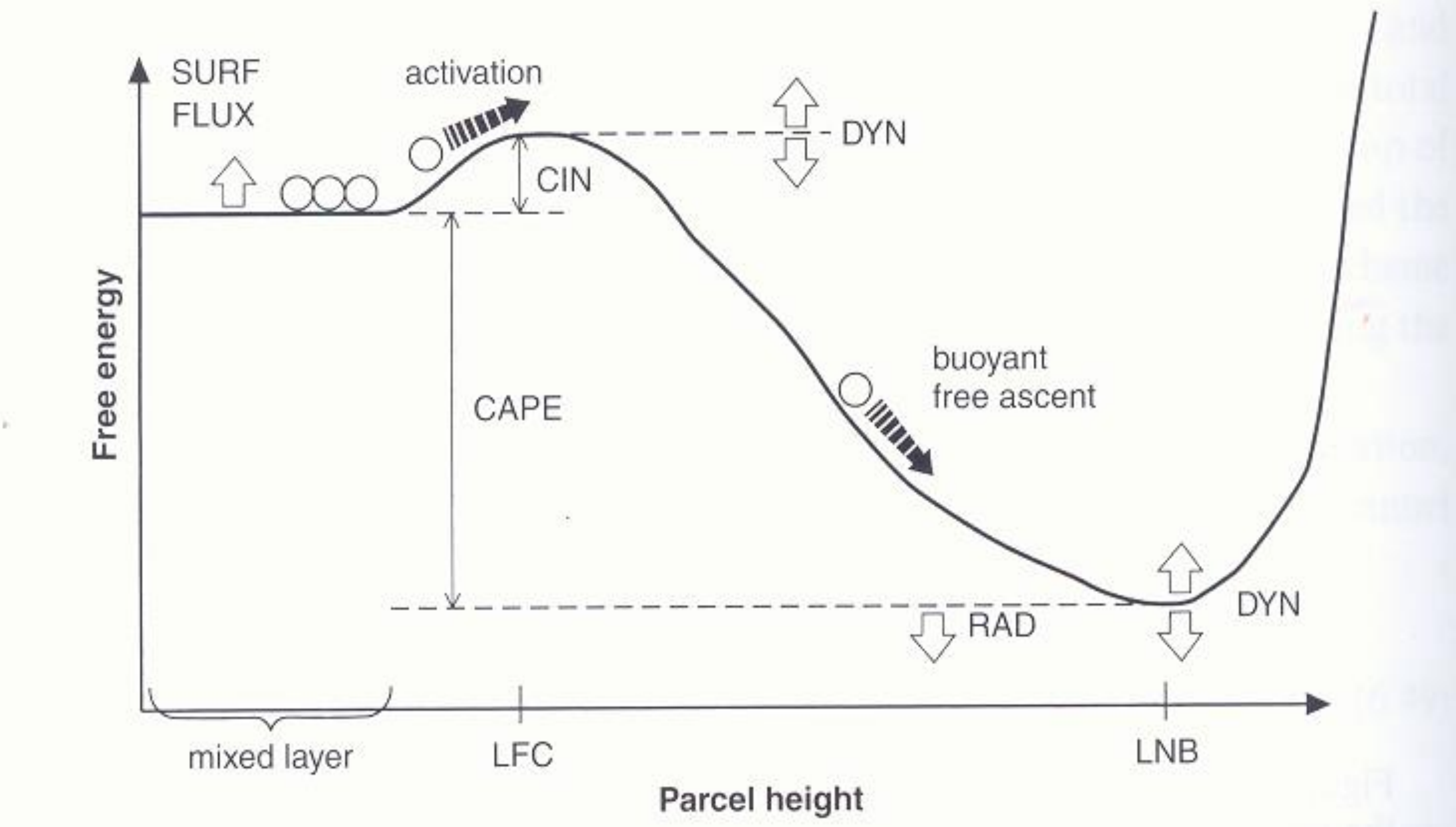
- The properties of the entrained air must, therefore, depend on which part of the variable environment in which an updraft happens to find itself. In addition, the representation of microphysical processes is extremely crude.

- The cloud dynamics is highly simplified in large-scale models.

UNCERTAINTIES IN FORMULATING CLOUD AND ASSOCIATED POCESSES



Arakawa 2004, Jour. Of Climate



A free energy diagram of a parcel model for deep convection; The vertical height of a parcel is indicated by the horizontal axis, with the level of free conv (LFC) and LNB indicated. The cumulative work done in lifting the parcel is indicated by the heavy solid line as a function of the parcel height. Parcels are indicated by circles and must overcome an energy barrier (CIN) to activate the CAPE. Dynamical processes (DYN) can vary CIN and CAPE while surf. Fluxes (SURF. FLUX) and radiation (RAD) change the amount of free energey (MAPES 1997)

Transport equation for a generic quantity X

SGS flux divergence

Source terms

$$\frac{\partial \overline{X}}{\partial t} + \frac{\partial\left(\overline{u_i}\,\overline{X}\right)}{\partial x_i} = \ldots - \frac{\partial \overline{u_i' X'}}{\partial x_i} + \overline{S}_x$$

Temperature and specific-humidity equations

$$\frac{\partial \overline{T}}{\partial t} + \frac{\partial\left(\overline{u_i}\,\overline{T}\right)}{\partial x_i} = \boxed{-\left.\frac{\partial \overline{u_i' T'}}{\partial x_i}\right|_{conv}} - \left.\frac{\partial \overline{u_i' T'}}{\partial x_i}\right|_{turb} - \left.\frac{\partial R_i}{\partial x_i}\right|_{rad} + \boxed{L(\overline{c} - \overline{e})\big|_{conv}} + L(\overline{c} - \overline{e})\big|_{grid-scale}$$

$$\frac{\partial \overline{q}}{\partial t} + \frac{\partial\left(\overline{u_i}\,\overline{q}\right)}{\partial x_i} = \boxed{-\left.\frac{\partial \overline{u_i' q'}}{\partial x_i}\right|_{conv}} - \left.\frac{\partial \overline{u_i' q'}}{\partial x_i}\right|_{turb} + \boxed{(\overline{e} - \overline{c})\big|_{conv}} + (\overline{e} - \overline{c})\big|_{grid-scale}$$

Here, L is the specific heat of vaporization, e is the rate of evaporation, and c is the rate of condensation.

Apart from mixing (redistribution of heat and moisture), convection produces precipitation

KUO Type convection (1965, JAS, Vol. 22, 40-63)

✓ The effect on large scale motions of latent heat release by deep cumulus convection in a conditionally unstable atmosphere

✓ It relates convective activity to total column moisture convergence, and come under deep-layer control scheme. It is a static scheme as it is not concerned with the details of convective processes and a moisture control scheme since it is closely tied to the available moisture.

✓They have shown that deep cumulus convective motions bring the moist surface air directly to higher levels, the time changes of temperature and mixing ratio can be determined from the horizontal advection of humidity and the vertical temperature and humidity distributions.

✓The derivation of the KUO scheme begins from the large scale equations in pressure co-ordinates (x,y,p) for the potential temperature and the water vapour mixing ratio with

Objectives of KUO paper

We have two main objectives in this paper: First we shall show that the statistical effect of the convective motions can be included without referring to their details by using a certain averaging process, and then we shall derive the formulas that express the latent heat released by the deep cumulus purely in terms of parameters of large scale quantities. This is accomplished through the application of a very simple cloud model, which is essentially the ejection of mean surface air from a source point. Secondly, we shall apply this method of including the latent heat released by deep cumulus convections to the investigation of the hurricane development problem, by incorporating this heat input in a simplified version of a time-dependent dynamic model and integrating it numerically.

Governing equations

Thus, the equations for the potential temperature θ, the water vapor mixing ratio q, and the horizontal velocity $\mathbf{V}$ of the large-scale system can be written in (x, y, p, t) coordinates as

$$\frac{d\bar{\theta}}{dt}-Q_r-\frac{L}{C_p}\bar{C}=\frac{L\pi}{C_p}\bar{C}^*-\frac{\partial\overline{\omega'\theta'}}{\partial p}-\nabla\cdot\overline{\mathbf{V}'\theta'}, \qquad (1)$$

$$\frac{d\bar{q}}{dt}+\bar{C}-T_q=-\bar{C}^*-\frac{\partial\overline{\omega'q'}}{\partial p}-\nabla\cdot\overline{\mathbf{V}'q'}, \qquad (2)$$

$$\frac{d\bar{\mathbf{V}}}{dt}+f\mathbf{k}\times\bar{\mathbf{V}}+\nabla\Phi-\mathbf{F}=\frac{\overline{\omega'\partial\mathbf{V}'}}{\partial p}-\overline{\mathbf{V}'\cdot\nabla\mathbf{V}'}, \qquad (3)$$

$$\chi=\bar{\chi}+\chi', \quad \bar{\chi}=\frac{1}{A}\int_A \chi\, dA, \quad \bar{\chi}'=0.$$

KUO scheme relates convective activity to total column moisture convergence

$$\frac{\partial \overline{\theta}}{\partial t} + \nabla \cdot (\overline{\vec{V}} \overline{\theta}) + \frac{\partial}{\partial p}(\overline{\omega}\overline{\theta}) = \frac{L_v(\overline{c} - \overline{e}) + Q_R}{\pi} - \frac{\partial}{\partial p}(\overline{\omega'\theta'}),$$

$$\frac{\partial \overline{q}}{\partial t} + \nabla \cdot (\overline{\vec{V}} \overline{q}) + \frac{\partial}{\partial p}(\overline{\omega q}) = -(\overline{c} - \overline{e}) - \frac{\partial}{\partial p}(\overline{\omega' q'}),$$

Q_R is the heating rate due to radiation, $\overline{c}$ is the rate of condensation per unit mass, $\overline{e}$ is the rate of evaporation per unit mass. Lv is latent heat of vaporization, q is the mixing ratio of water vapour, π is the exner function.

where $\bar{C}$ and C^* are the condensation rates produced by the large-scale motions and by the subgrid-scale convective motions, respectively, L is the latent heat of condensation, Q_r the heating rate by radiation and turbulent diffusion, T_q and $\mathbf{F}$ the rates of turbulent diffusion of moisture and momentum, and d/dt the rate of change observed by following the large-scale flow, *viz.*

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla+\bar{\omega}\frac{\partial}{\partial p},$$

and $\pi=(P/p)^{R/C_p}$. The other symbols have their usual meanings. Here we have put all the contributions from the subgrid-scale flows to the right-hand side of the equations.

As in the author's earlier work, we shall use the net convergence of moisture into the vertical column of air of unit cross section produced by the large-scale flow and by evaporation from the ground as one fundamental parameter. Denoting this quantity by M_t, we then have

$$M_t = -\frac{1}{g}\int_0^{p_0} (\nabla \cdot \overline{V}\bar{q})dp + \rho_0 C_D V_0 (q_g - q_0), \quad (4)$$

where q_g is the value of q at the surface, q_0 is that at a nearby level, and C_D is the drag coefficient.

We assume that a fraction $(1-b)$ of the total convergence of moisture M_t is condensed and either precipitated out as rain or carried away, while the remaining fraction b of M_t is stored in the air to increase the humidity, including the influence of evaporation of condensed water. That is to say, we have

$$PR = (1-b)\left[-\frac{1}{g}\int_0^{p_{sfc}} \nabla \cdot (\overrightarrow{V}\overline{q})\, dp + \frac{1}{L_v} Q_E\right] = (1-b)M_t,$$

$$M = (1-b)M_t \left\{ \begin{array}{l} \text{[precipitated or carried} \\ \text{away part of the mois-} \\ \text{ture convergence]} \end{array} \right. \qquad (5)$$

$$\frac{1}{g}\int_0^{p_s} \frac{\partial q}{\partial t} dp = bM_t. \qquad (6)$$

We expect b to be much smaller than 1 in the regions of low-level convergence in the tropics. The vertical distribution of this part is given by $(q_s - \overline{q})$.

- Q_E is the latent heat flux, b is a constant

- The total heating that is released by latent heating by deep cumulus

$$aQ_c = \frac{g(1-b)LM_t}{C_p(p_b - p_t)} \frac{(\theta_c - \bar{\theta})\pi}{\langle \theta_c - \theta \rangle}$$

eating that convection produces can be written as

$$\frac{1}{g}\int_0^{p_{sfc}} L_v(\bar{c} - \bar{e})\, dp - \frac{1}{g}\int_0^{p_{sfc}} \pi \frac{\partial}{\partial p}(\overline{\omega'\theta'})\, dp = L_v(1-b)M_t + Q_H,$$

(7)

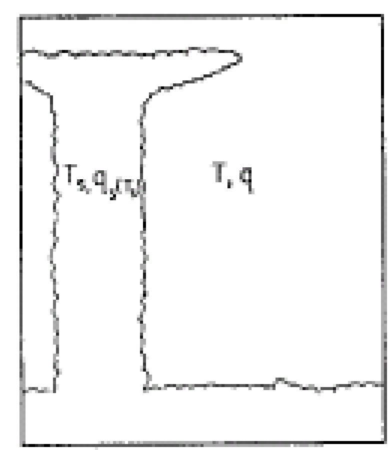
In order to be definite we make the following simple assumptions concerning such deep cumulus clouds:

i) Cumulus convection always occurs in regions of deep layers of conditionally unstable stratification and mean low level convergence.

ii) Such convective motions bring surface air to all levels up to a great height so that inside the cloud the vertical distributions of temperature and mixing ratio are those of the moist adiabat through the appropriate condensation level.

iii) The base of the cloud is at the condensation level of the surface air and the top extends to the level where the moist adiabat through the condensation level meets the environmental temperature profile, or somewhat higher.

iv) The cumulus clouds exist only momentarily. They dissolve by mixing with the environmental-air at the same level, so that the heat and moisture carried up by the cloud air are imparted to the environmental air.



Kuo Scheme: Description, Models, & Trigger

Description: This is a simple scheme that produces precipitation and increases static stability by emulating the moist-adiabatic ascent of a parcel. It adjusts the temperature and moisture profiles toward moist adiabatic.

Convective Process:

Trigger: Convection is triggered by any amount of CAPE and column-integrated moisture convergence exceeding a threshold value.

Mass-Flux Schemes. Basic Features

A triple top-hat decomposition

$$\overline{X} = a_u X_u + a_d X_d + a_e X_e, \quad a_u + a_d + a_e = 1,$$

"u", "d" and "e" refer to the updraught, downdraught and the environment, respectively, and a is the fractional area coverage.

In terms of the probabilities (δ is the Dirac delta function)

$$\overline{X} = P_u X_u + P_d X_d + P_e X_e, \quad P(X') = P_u \delta(X' - X_u) + P_d \delta(X' - X_d) + P_e \delta(X' - X_e).$$

Vertical flux of a fluctuating quantity X

$$\begin{aligned}\overline{\rho}\overline{w'X'} &= \overline{\rho} a_u (w_u - \overline{w})(X_u - \overline{X}) + \overline{\rho} a_d (w_d - \overline{w})(X_d - \overline{X}) + \overline{\rho} a_e (w_e - \overline{w})(X_e - \overline{X}) \\ &= M_u (X_u - \overline{X}) + M_d (X - X_d) + M_e (X_e - \overline{X}),\end{aligned}$$

$M_u = \overline{\rho} a_u (w_u - \overline{w})$ is the updraught mass flux (similarly for downdraught and environment).

Assumption 1: a mean over the environment is equal to to a horizontal mean (over a grid box),

$$X_e = \overline{X}, \qquad a_u \ll 1 \quad and \quad a_d \ll 1.$$

Assumption 2: convection is in a quasi-steady state,

$$\left(\frac{\partial}{\partial t} + \overline{w}\frac{\partial}{\partial z}\right)a_u = 0, \quad \left(\frac{\partial}{\partial t} + \overline{w}\frac{\partial}{\partial z}\right)(a_u X_u) = 0.$$

Then, vertical flux of a fluctuating quantity X in mass-flux approximation is given by

$$\overline{w'X'} = \frac{1}{\overline{\rho}}\left[M_u X_u + M_d X_d - (M_u + M_d)\overline{X}\right]$$

Kuo Scheme: Strengths & Limitations

Strengths

- Essence and behavior is easy to understand
- Runs quickly; does not require much computing resources

Limitations

- Simplistic scheme; cannot represent the variety of things that happen in nature
- Does not account for the strength of cap inhibiting convective development
- Positive feedback (including precipitation bull's-eyes) sometimes occurs because the model response to parameterized convective heating may generate moisture convergence, which triggers the scheme again. This behavior stems from assuming that moisture convergence causes convection
- Many variations exist (for example, some include downdrafts, while others do not). Each formulation results in a variety of unrealistic physical behaviors

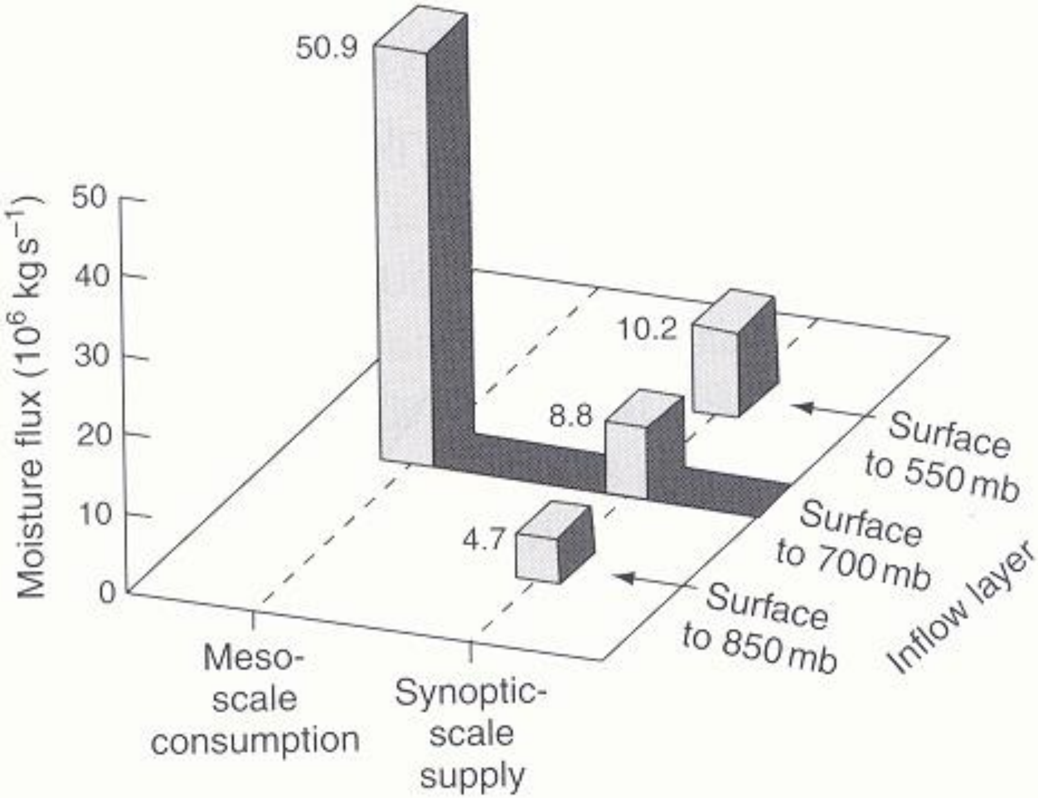


Figure 6.23. Comparison of synoptic-scale moisture flux into a squall line with the mesoscale moisture flux of the squall line for a grid-sized atmospheric column in northern Oklahoma. These consumption and supply terms should roughly balance if the Kuo closure is reasonable. From Fritsch *et al*. (1976).

Chapter 10

The Arakawa–Schubert Cumulus Parameterization

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10.1. Introduction

The motivation of the paper by Arakawa and Schubert (1974) (hereafter referred to as A–S) was to present a theoretical framework that can be used for understanding the physical and logical basis for cumulus parameterization. More specifically, the paper attempted to answer the following questions:

1) How can cumulus clouds modify their environment while condensation takes place only inside clouds. Does the modification occur only through mixing of cloud air with the environment as clouds decay? If not, how can clouds in their mature phase modify the environment?

2) Since the vertical structure of that modification depends on cloud type (primarily cloud-top height) and typically more than one type of clouds coexist, a parameterization scheme should determine the spectral distribution of clouds instead of assuming a particular cloud type (or particular cloud types) a priori. What is an appropriate framework for doing this?

3) How does the subcloud layer control cumulus activity? What is the nature of the feedback in this link?

4) Finally, how can the intensity of overall cumulus activity and that of each cloud type be determined for a given large-scale condition? It is obvious that we can only parameterize cumulus activity in balance with large-scale processes. Then, how can we quantitatively define such balance?

The parameterization by Arakawa (1969) presents preliminary answers to these questions. It was designed for a three-layer (or three-level) model, in which the lowest layer represents the planetary bounday layer (PBL). Corresponding to this vertical structure, three types of clouds are considered: deep and shallow clouds originating from the PBL and "middle-level" clouds originating above the PBL. Despite its simplified vertical structure, this parameterization explicitly formulates the effects of cloud-induced subsidence and cloud air detrainment on the large-scale environment. For type I closure (see chapter 1), the parameterization introduces a measure of the bulk buoyancy of cloud air, given by the difference between lower-level moist static energy and upper-level saturation moist static energy. When large-scale processes tend to increase the bulk buoyancy for a certain cloud type or cloud types, cloud-base mass flux is determined for each cloud type to restore the bulk buoyancy toward an equilibrium.

The A–S parameterization generalizes and elaborates the basic ideas of Arakawa (1969) described above. Sections 10.2, 10.3, 10.4, and 10.5 describe how the parameterization addresses questions 1, 2, 3, and 4, respectively. Section 10.6 outlines standard procedures in practical application of the parameterization. Section 10.7 gives further comments. The Appendix describes an important recent revision of the parameterization through the inclusion of convective-scale downdraft effects. Readers are encouraged to read chapter 1 by Arakawa on closure assumptions as an introduction to this chapter.

10.2. Modification of the environment by cumulus clouds

This part of the A–S parameterization formulates the vertical distributions of cumulus heating and moistening in terms of the vertical distributions of mass flux through clouds, mass detrainment from clouds, and thermodynamical properties of detraining cloud air.

Let us consider an ensemble of clouds in a "large-scale" area, as shown in Fig. 10.1. The area is assumed to be large enough so that the cloud ensemble can be considered as a statistical entity but small enough so that the cloud environment is approximately uniform horizontally. The existence of such an area is an idealization. In practice, cloud effects averaged over the area may not be statistically significant, especially when the system.

Since cumulus parameterization is an attempt to formulate the collective effect of cumulus clouds without predicting individual clouds, it is a *closure problem* in which we seek a limited number of equations that govern the statistics of a system with huge dimensions. The core of the parameterization problem is, therefore, in the choice of appropriate closure assumptions. When we have global models with comprehensive physics in mind, rather than idealized models with a more limited scope, closure assumptions must meet the following requirements:

(i) *Closure assumptions must not lose the predictability of large-scale fields.* This is an obvious requirement since we need to parameterize clouds for predicting the time evolution of large-scale fields. If we wish to assume that a certain variable is in an equilibrium, the variable must be one whose prediction is not intended by the model.

(ii) *Closure assumptions must be valid quasi-universally.* This is also an obvious requirement because comprehensive global models must be valid for a variety of synoptic and surface conditions.

One may then ask, Can we really find closure assumptions satisfying these requirements? In other words, To what extent is it possible to parameterize cumulus clouds? These are difficult questions to answer. The difficulty is amplified by the existence of intermediate scales in cloud organization, which are gen-

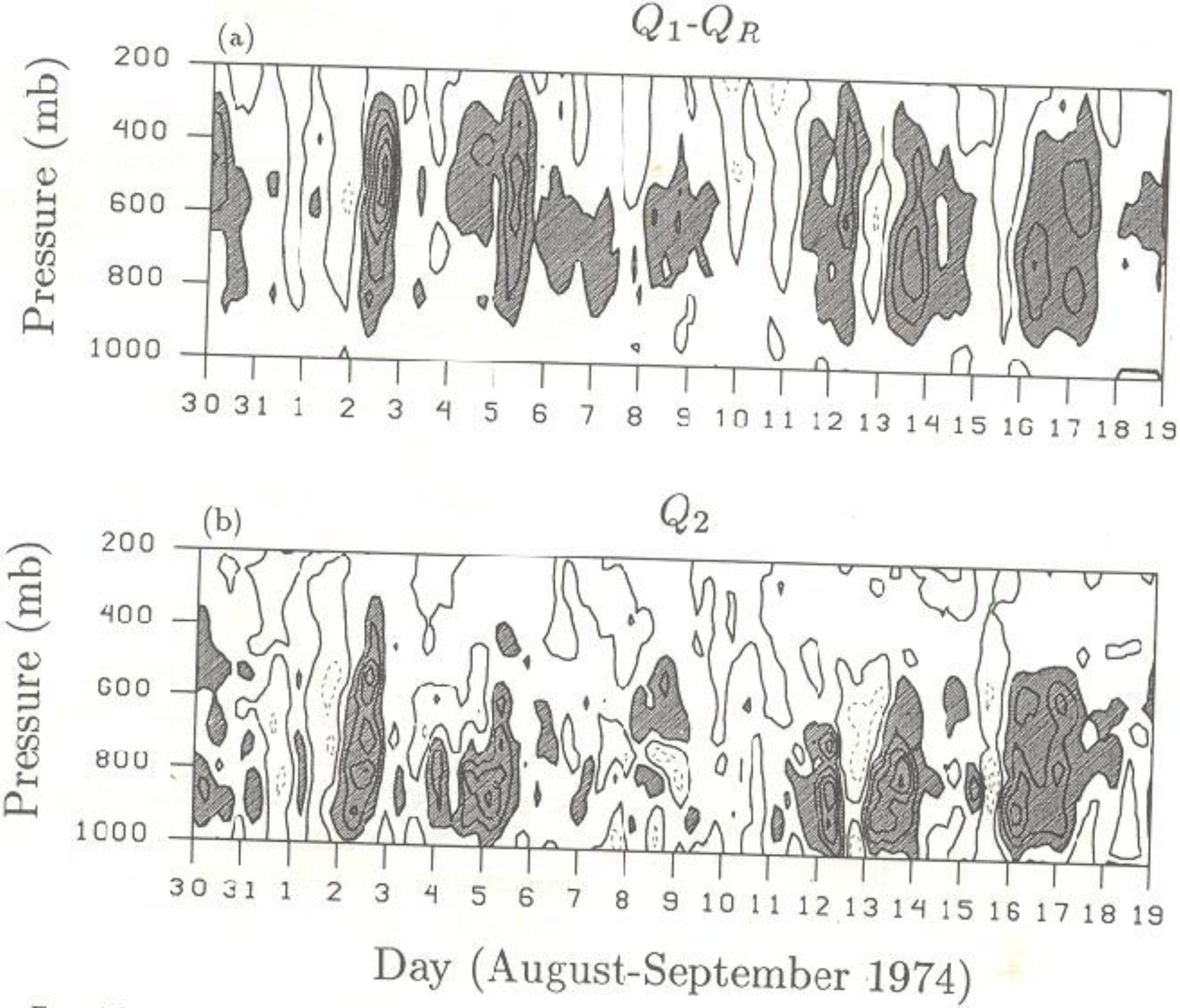


FIG. 1.2. Diagnostically calculated (a) Q_{1C} ($\equiv Q_1 - Q_R$) and (b) Q_2 for the GATE phase III period averaged over the 3° × 3° grid box centered at 8.5°N, 23.5°W. Contour intervals are equivalent to 5°C day^{-1} when divided by c_p. Negative values are shown by dashed lines, and areas representing values larger than 5°C day^{-1} are shaded; redrawn from Arakawa and Chen (1987).

1.3. A classification of closure assumptions

In this section we classify closure assumptions into basic types. For convenience, we rewrite (1.1) and (1.2) as

$$\frac{\partial T}{\partial t} = \left(\frac{\partial T}{\partial t}\right)_{\mathrm{LS}} + \frac{1}{c_p} Q_1, \tag{1.7}$$

$$\frac{\partial q}{\partial t} = \left(\frac{\partial q}{\partial t}\right)_{\mathrm{LS}} - \frac{1}{L} Q_2. \tag{1.8}$$

Here and throughout the rest of this chapter, overbars for area average are omitted. The subscript LS denotes the contribution to the time derivative by large-scale advective processes: that is,

$$\left(\frac{\partial T}{\partial t}\right)_{\mathrm{LS}} = -\left(\frac{p}{p_0}\right)^{R/c_p}\left(\bar{\mathbf{v}} \cdot \nabla \bar{\theta} + \bar{\omega} \frac{\partial \bar{\theta}}{\partial p}\right), \tag{1.9}$$

$$\left(\frac{\partial q}{\partial t}\right)_{\mathrm{LS}} = -\left(\bar{\mathbf{v}} \cdot \nabla \bar{q} + \bar{\omega} \frac{\partial \bar{q}}{\partial p}\right). \tag{1.10}$$

In the diagnosis of Q_1 and Q_2 from observations hown in section 2, we treated the tendency terms as nown quantities. In the parameterization problem,

Arakawa-Schubert , 1974, JAS, 674-701

A theory of the interaction of a cumulus cloud ensemble with the large-scale environment is developed.

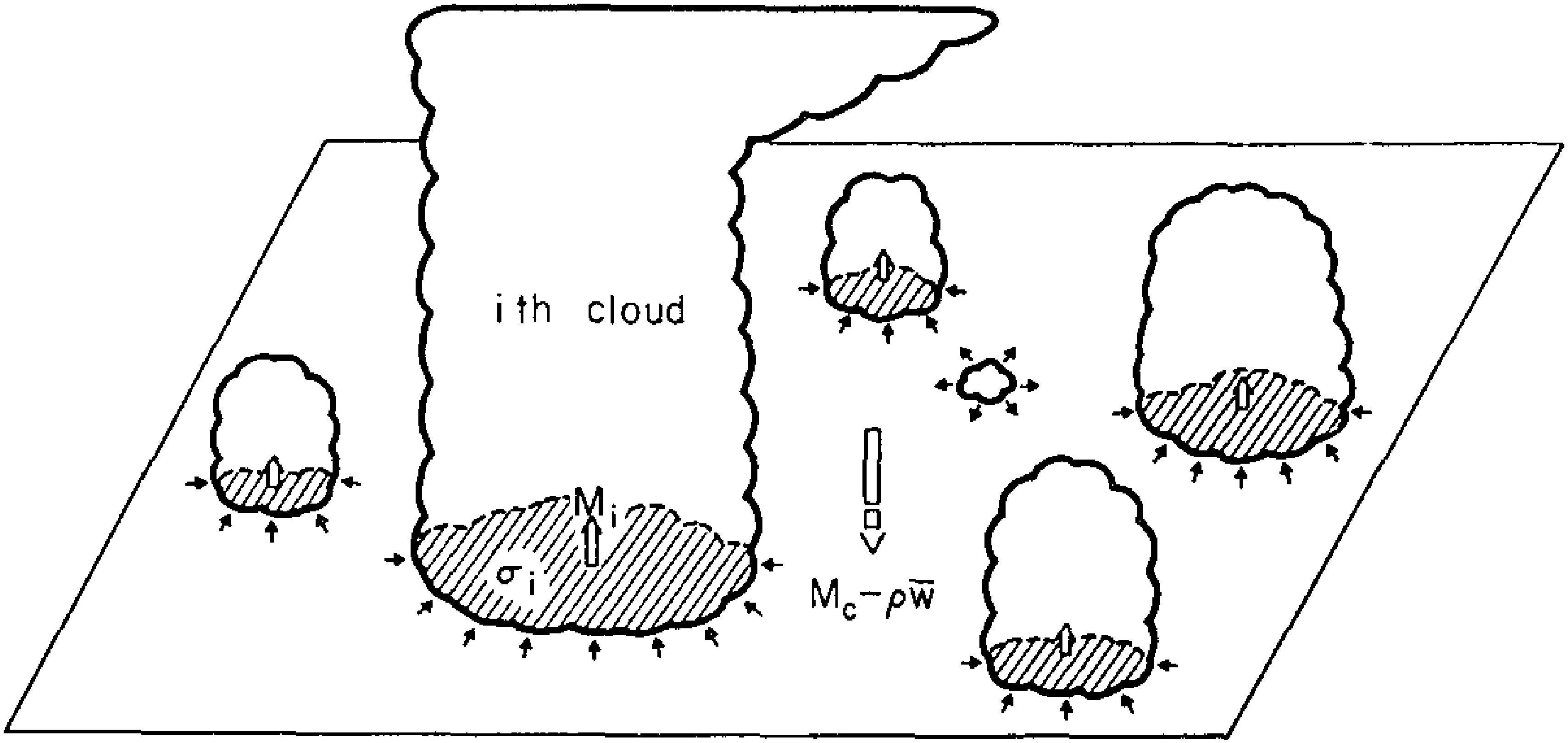
In this theory, the large-scale environment is divided into the subcloud mixed layer and the region above.

The time changes of the environment are governed by the heat and moisture budget equations for the subcloud mixed layer and for the region above, and by a prognostic equation for the depth of the mixed layer.

In the environment above the mixed layer, the cumulus convection affects the temperature and moisture fields through cumulus-induced subsidence and detrainment of saturated air containing liquid water which evaporates in the environment. In the subcloud mixed layer, the cumulus convection does not act directly on the temperature and moisture fields, but it affects the depth of the mixed layer through cumulus-induced subsidence. Under these conditions, the problem of parameterization of cumulus convection reduces to the determination of the vertical distributions of the total vertical mass flux by the ensemble, the total detrainment of mass from the ensemble, and the thermodynamical properties of the detraining air.

The cumulus ensemble is spectrally divided into sub-ensembles according to the fractional entrainment rate, given by the ratio of the entrainment per unit height to the vertical mass flux in the cloud. For these sub-ensembles, the budget equations for mass, moist static energy, and total water content are obtained. The solutions of these equations give the temperature excess, the water vapor excess, and the liquid water content of each sub-ensemble, and further reduce the problem of parameterization to the determination of the mass flux distribution function, which is the sub-ensemble vertical mass flux at the top of the mixed layer.

The cloud work function, which is an integral measure of the buoyancy force in the clouds, is defined for each sub-ensemble; and, under the assumption that it is in quasi-equilibrium, an integral equation for the mass flux distribution function is derived. This equation describes how a cumulus ensemble is forced by large-scale advection, radiation, and surface turbulent fluxes, and it provides a closed parameterization of cumulus convection for use in prognostic models of large-scale atmospheric motion.



A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

The horizontal area must be large enough to contain an ensemble of cumulus cloud but small enough to cover only a fraction of large scale disturbance. The existence of such an area is one of the basic assumptions of this paper

As acoustic waves are not of concern, the mass continuity equation in quasi-Boussinesq form

$$\vec{\nabla}.(\rho\vec{V}) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (1)$$

Density ρ is a function of height only, V is the horizontal velocity, $\vec{\nabla}$ is horizontal del operator

W is the vertical velocity and z the vertical coordinate.

Let $\sigma_i(z,t)$ be the fractional area covered by the ith cloud, in a horizontal cross section at level z and time t.

The vertical mass flux through σ_i is

$$M_i = \int_{\sigma_i} \rho w d\sigma = \rho \sigma_i w_i,$$

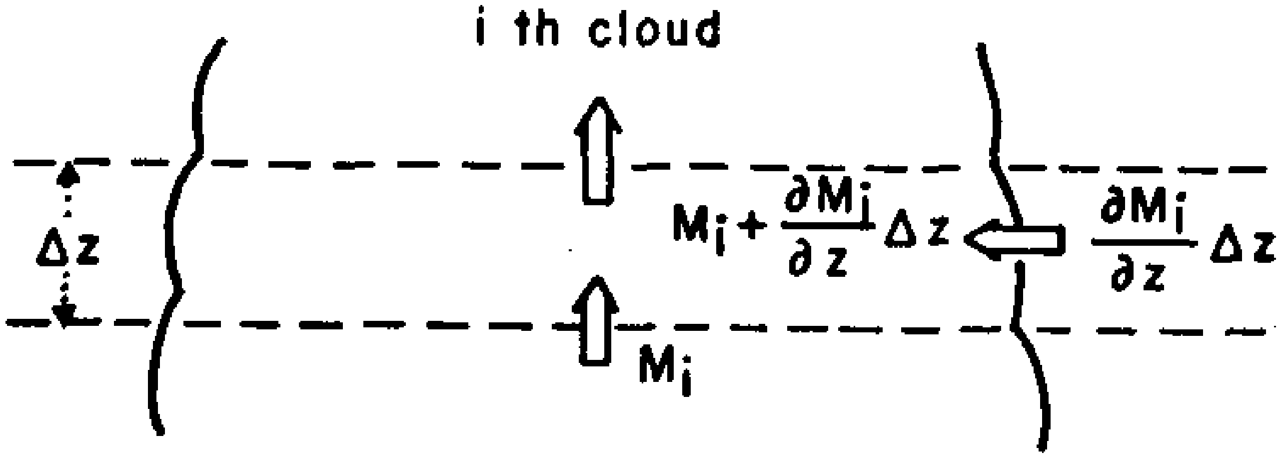
where

$$\int_{\sigma_i} d\sigma$$

- **Trigger:**
- To trigger convection, the scheme requires some boundary-layer CAPE.

- Although it varies in specific implementations, the general formulation requires the presence of large-scale atmospheric destabilization with time. The process by which the scheme attempts to assess destabilization is complex; for example, it must account for the effects of entrainment and clouds of various depths.

The inward mass flux per unit height, normal to the lateral boundary of the ith cloud, is given by $\partial M_i/\partial z$



A schematic diagram of the mass continuity for a thin layer in the ith cloud.

Here the boundary is not necessarily vertical. Then the mass added to the cloud, which may be horizontally expanding or shrinking, is $\partial M_i/\partial z + \rho\partial\sigma_i/\partial t$ per unit height and unit time. The entrainment and detrainment of mass are given by

Entrainment:

$$E_i=\left(\frac{\partial M_i}{\partial z}+\rho\frac{\partial \sigma_i}{\partial t}\right), \quad \text{when} \quad \frac{\partial M_i}{\partial z}+\rho\frac{\partial \sigma_i}{\partial t}>0$$

Detrainment:

$$D_i=-\left(\frac{\partial M_i}{\partial z}+\rho\frac{\partial \sigma_i}{\partial t}\right), \quad \text{when} \quad \frac{\partial M_i}{\partial z}+\rho\frac{\partial \sigma_i}{\partial t}<0.$$

E_i can be rewritten as

$$\text{As } M_i = \rho \sigma_i w_i$$

$$\frac{\partial M_i}{\partial z} + \rho \frac{\partial \sigma_i}{\partial t} \text{ can be rewritten as}$$

$$\frac{\partial (\rho \sigma_i w_i)}{\partial z} + \rho \frac{\partial \sigma_i}{\partial t}$$

$$E_i = \underbrace{\sigma_i \frac{\partial (\rho w_i)}{\partial z}}_{\text{Mass flux}} + \underbrace{\rho \left(\frac{\partial}{\partial t} + w_i \frac{\partial}{\partial z}\right) \sigma_i}_{\text{Expansion of cloud}}$$

Thus entrainment of mass which is caused by turbulent mixing at the cloud boundary appears either as a vertical divergence of mass flux within the cloud, as a horizontal expansion of the cloud as it rises or as a combination of these two depending on the dynamics of the clouds.

The total vertical mass flux by all of the clouds in the ensemble is

$$\text{As } M_i = \rho\sigma_i w_i$$

$$M_c = \sum_i M_i$$

where $\sum_i$ denotes the summation over all clouds which are penetrating the levels being considered

Let $\rho\overline{w}$ be the net vertical mass flux over the large scale unit horizontal area. It satisfies the continuity equation

$$\overline{\vec{\nabla}.\rho\vec{v}} + \frac{\partial}{\partial z}(\rho\overline{w}) = 0$$

where the bar denotes the average over the unit horizontal area

In general the total vertical mass flux is M_c in the clouds is not the same as the large scale net vertical mass flux through the unit large scale horizontal area $\rho\bar{w}$. The difference between Mc and ρw is equal to the downward mass flux between the clouds

$$\tilde{M} = \rho\bar{w} - M_c$$

with sufficiently intense cumulus activity M_c can exceed

$\rho\bar{w}$ and subsidence (negative $\tilde{M}$) appears in the environment.

At a given height some clouds may be detraining and some others are entraining. Total entrainment and total detrainment are defined as E and D respectively.

$$E = \sum_{e.c} E_i$$

$$D = \sum_{d.c} D_i$$

Here $\sum_{e.c}$ denotes the summation over all clouds which are entraining at that level

E, D and M_c are functions of z

$$E - D = \frac{\partial M_c}{\partial z} + \rho \frac{\partial \sigma_c}{\partial t}$$

~ deonotes a value
In the env. Overbar
Denotes ave over
Large scale area

The total mass flux in the clouds, M_c, can be expressed as

$$M_c(z) = \int_0^{\lambda_D(z)} \mathfrak{M}(z,\lambda)\,d\lambda, \tag{77}$$

where

$$\mathfrak{M}(z,\lambda)\,d\lambda = \sum_{\lambda_i \epsilon (\lambda,\lambda+d\lambda)} M_i(z) \tag{78}$$

is the sub-ensemble mass flux due to the clouds which have the parameter λ_i in the interval $(\lambda, \lambda+d\lambda)$.

The total detrainment $D(z)dz$ in the layer between z and $z+dz$ is equal to the sub-ensemble mass flux, at level z, due to the clouds which have parameter λ_i in the interval $\lambda_D(z)-(-d\lambda_D(z)/dz)dz$ to $\lambda_D(z)$, as is shown in Fig. 3. Then we have

$$D(z)=-\mathfrak{M}(z,\lambda_D(z))\frac{d\lambda_D(z)}{dz}.$$

It is convenient to normalize $\mathfrak{M}(z,\lambda)$ by

$$\mathfrak{M}(z,\lambda)\equiv\mathfrak{M}_B(\lambda)\eta(z,\lambda),$$

where

$$\mathfrak{M}_B(\lambda)\equiv\mathfrak{M}(z_B,\lambda),$$

Our final problem is to find the mass flux distribution function, $\mathcal{M}_B(\lambda)$. The real conceptual difficulty in parameterizing cumulus convection starts from this point. We must determine how the large-scale processes control the spectral distribution of clouds, in terms of the mass flux distribution function, if they indeed do so at all. This is the essence of the parameterization problem.

The solution for $\mathcal{M}_B(\lambda)$ may be even more difficult than just the determination of the predominant cloud size; instead, we must determine the entire spectrum of the clouds. But, on the other hand, in a parameterization theory it is necessary to find only the statistical properties of the cumulus ensemble, under given large-scale conditions, and not the properties of each individual cloud at a given place and time. Also, with the approximations that are used in this parameterization theory, we need to obtain only the mass flux distribution function, $\mathcal{M}_B(\lambda)$, and not necessarily the population distribution in λ space. These two are generally not equivalent.

The time change of the kinetic energy of each sub-ensemble can be written as

$$\frac{d\mathcal{K}(\lambda)}{dt} = A(\lambda)\mathcal{M}_B(\lambda) - \mathcal{D}(\lambda), \qquad (132)$$

$A(\lambda)$ the "cloud work function."

$A(\lambda)$ is the kinetic energy generation per unit mass flux, it is a measure of the efficiency of the kinetic energy generation. It is given by

$$A(\lambda) = \int_{z_B}^{z_D(\lambda)} \frac{g}{c_p \bar{T}(z)} \eta(z,\lambda)[s_{vc}(z,\lambda) - \bar{s}_v(z)]dz, \qquad (133)$$

Since individual cumulus clouds are necessarily subgrid-scale in any numerical model of large-scale circulations, the collective effects of clouds within a model grid box must be expressed, or parameterized, in terms of the grid-point variables. Although it is not completely obvious that this goal may be attained, the frequent organization of the individual clouds into clusters indicates that they are statistically coupled with the large-scale dynamical and thermodynamical processes.

We define the goal of a cumulus parameterization for large-scale numerical prediction models as

follows: to *predict* changes in the grid-scale variables due to subgrid-scale cumulus convection. *Since, as stated earlier, the collective effects are* to be parameterized, an overall measure of the intensity of subgrid-scale cumulus convection must *be determined from the grid-scale variables. This* determination is impossible without a closure assumption and, therefore, no parameterization is complete without one.

In order for a cumulus parameterization to be feasible, some kind of statistical balance must exist between *the cumulus cloud ensemble and the grid-*scale variables. If such balance does not exist it is in principle impossible to parameterize cumulus convection. *Any* closure assumption for cumulus parameterization, therefore, can be interpreted as an assumed balance between the cumulus cloud ensemble and *the* grid-scale variables. *It is* very important that the assumed balance be explicitly stated as a closure assumption so that it can be rationally evaluated.

A theory of the interaction of a cumulus ensemble with the large-scale environment was fully described in Part I. Cumulus clouds are assumed to modify the large-scale environment by compensating subsidence between the clouds and by detrainment of cloud air containing suspended liquid water droplets. Since the cumulus parameterization must predict the vertical distribution of cumulus modification of the large-scale environment, the cumulus cloud *ensemble is divided into* subensembles according to a spectral parameter λ. The fundamental assumption in the spectral decomposition *of the cloud ensemble is that* λ characterizes the statistical properties of all members of the subensemble and consequently all subensemble members *modify the environment in the same manner.* The specific choice for λ is discussed in the Appendix.

and water vapor mixing ratio. Therefore, the parameterization of cumulus convection is reduced to the determination of the remaining unknown, the distribution of cloud-base mass flux $\mathcal{M}_B(\lambda)$.

As discussed in the Introduction, a closure assumption is needed to complete any cumulus parameterization. In the Arakawa-Schubert parameterization, the closure takes the form of a balance between the generation of moist convective instability by the large-scale processes and its destruction by clouds as shown schematically in Fig. 1. Cloud-scale kinetic energy is the manifestation of a moist convective instability in the environment.

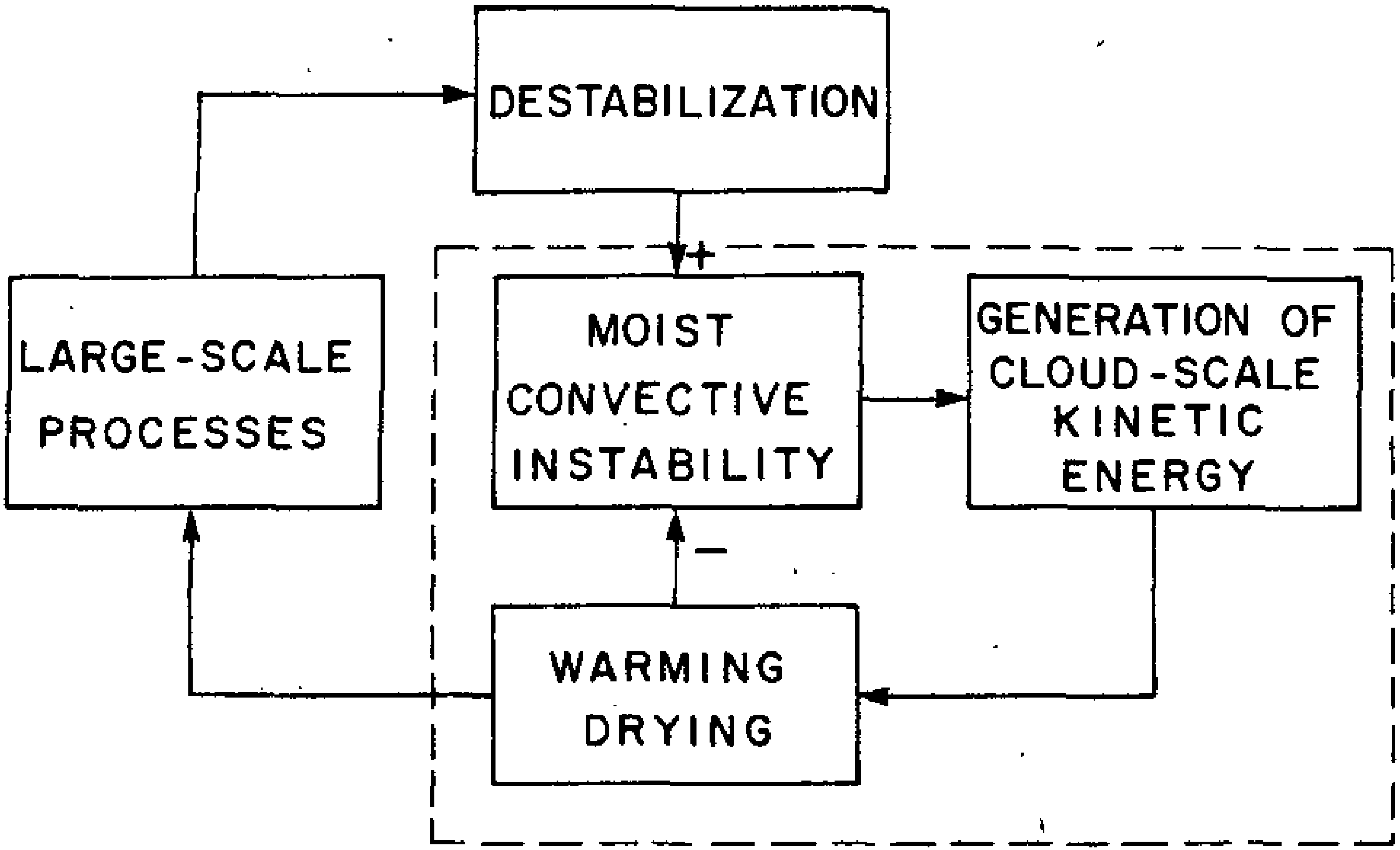


FIG. 1. A schematic diagram of the Arakawa-Schubert closure assumption. The dashed box represents the cumulus parameterization.

Let us now examine the cumulus subensemble kinetic energy budget to provide a physical interpretation of this closure assumption. For a cloud subensemble with fractional entrainment rate between λ and $\lambda + d\lambda$, let $\mathcal{K}(\lambda)d\lambda$ be the cloud-scale kinetic energy for the subensemble, $A(\lambda)$ be the cloud-work function defined as the cloud-scale kinetic energy generation per unit $\mathcal{M}_B(\lambda)d\lambda$, and $\mathcal{D}(\lambda)$ be the cloud-scale kinetic energy dissipation per unit $\mathcal{M}_B(\lambda)d\lambda$. The kinetic energy budget for the cloud subensemble may be written as

$$\frac{d}{dt}\mathcal{K}(\lambda)d\lambda = [A(\lambda) - \mathcal{D}(\lambda)]\mathcal{M}_B(\lambda)d\lambda. \qquad (1)$$

Now consider a hypothetical situation in which there is no generation of kinetic energy and, therefore, only dissipation by various processes is acting on the subensemble. Let τ_{DIS} be the decay time in this situation. Then, in terms of orders of magnitude, Eq. (1) gives

$$\mathscr{D}(\lambda)\mathscr{M}_B(\lambda) \sim \frac{\mathscr{K}(\lambda)}{\tau_{\mathrm{DIS}}} . \tag{2}$$

Let τ be the time scale over which we apply (1). Then using (2) in (1) we have

$$\frac{\mathscr{K}(\lambda)}{\tau} \sim A(\lambda)\mathscr{M}_B(\lambda) - \frac{\mathscr{K}(\lambda)}{\tau_{\mathrm{DIS}}} . \tag{3}$$

When $\tau \gg \tau_{\mathrm{DIS}}$, the left-hand side (lhs) of (3), and therefore that of (1), can be neglected. Eq. (1) then gives

$$A(\lambda) \approx \mathscr{D}(\lambda) \quad \text{for} \quad \mathscr{M}_B(\lambda) > 0. \tag{4}$$

This equation is a statement of the "kinetic energy quasi-equilibrium" for each cumulus subensemble.

Eq. (4) was derived on the assumption that $\tau_{DIS} \ll \tau$. By definition, τ_{DIS} is the time scale when only dissipative processes are acting; therefore, it must be smaller than an actual cloud lifetime which includes both generation and dissipation. We thus estimate τ_{DIS} to be of order 10^2-10^3 s. Since we wish to predict changes of a cumulus ensemble over the time scale of large-scale disturbances (τ_{LS}) an appropriate choice for τ is $\tau = \tau_{LS}$, where τ_{LS} is typically of order 10^5 s. Then $\tau_{DIS} \ll \tau_{LS}$ and, therefore, the kinetic energy quasi-equilibrium is a very good approximation.

tions. Therefore, in Part I, the cloud-work function $A(\lambda)$ was defined as the subensemble kinetic energy generation (per unit cloud-base mass flux) due to work done by the buoyancy force, i.e.,

$$A(\lambda) = \int_{z_B}^{\hat{z}(\lambda)} \frac{g}{\bar{T}(z)} \eta(z,\lambda)[T_{vc}(z,\lambda) - \bar{T}_v(z)]dz, \quad (5)$$

where $T_{vc}(z,\lambda)$ and $\bar{T}_v(z)$ are the subensemble and environmental virtual temperatures, $\hat{z}(\lambda)$ is the subensemble cloud-top height, and $\eta(z,\lambda)$ is the subensemble normalized vertical mass flux.

It is important to note that *for a given* λ, $A(\lambda)$ *depends solely on the large-scale thermodynamical vertical structure* since the difference $T_{vc}(z,\lambda) - \bar{T}_v(z)$ is determined by the vertical structure. It was shown in Part I that $A(\lambda)$ is a generalized measure of the moist convective instability in the large-scale environment.

With this definition of $A(\lambda)$, Eq. (4) has a clear physical interpretation. If the cloud-scale kinetic energy generation by the buoyancy force is more than is needed to balance the dissipation in a statistical sense, the vertical mass flux of clouds and, correspondingly, the induced subsidence between clouds will increase. The resulting increase in the warming and drying in the environment and decrease of the subcloud layer depth will tend to reduce the moist convective instability by decreasing the buoyancy felt by the clouds. Consequently, the cumulus-scale kinetic energy generation will decrease. Thus kinetic energy generation tends to balance dissipation, $A(\lambda) \approx \mathcal{D}(\lambda)$. If $A(\lambda) < \mathcal{D}(\lambda)$ there can be no sustained convection over the large-scale area. Therefore, when $A(\lambda) < \mathcal{D}(\lambda)$, $\mathcal{M}_B(\lambda) = 0$ and (4) does not apply. This situation has been

In general, dissipation in clouds should depend primarily on momentum entrainment through cloud boundaries and downward drag due to precipitation falling within the cumulus updrafts. Regardless of the details of the dissipation mechanisms, however, the total dissipation should be roughly proportional to the cloud mass flux. Then $\mathscr{D}(\lambda)$, which is the dissipation *per unit cloud-base mass flux*, will not depend substantially on the large-scale situation once the cloud type (i.e., λ) is specified. In other words, to a first order of approximation, $\mathscr{D}(\lambda)$ can be regarded as an intrinsic cloud subensemble property and hence is a quasi-constant for each cloud type.

We have shown that for a given λ, $A(\lambda)$ depends solely on the large-scale thermodynamical vertical structure. On the other hand, $\mathscr{D}(\lambda)$ is an intrinsic cloud subensemble property. The kinetic energy quasi-equilibrium (4) thus hypothesizes a remarkable relationship between the large-scale thermodynamical vertical structure and the cloud-scale dissipation. This relationship clearly depends on the large-scale vertical structure of both temperature *and* moisture. Therefore, the kinetic energy quasi-equilibrium regulates a coupled temperature and moisture structure in the large-scale environment. It does *not* imply that temperature and moisture are constrained individually.

To derive a practicable closure assumption, we take the derivative of (4) with respect to a time long enough for the kinetic energy quasi-equilibrium to hold. Then

$$\frac{d}{dt} A(\lambda) \approx \frac{d}{dt} \mathscr{D}(\lambda). \tag{6}$$

Based on the above argument, we may set

$$\frac{d}{dt} \mathscr{D}(\lambda) \approx 0, \tag{7}$$

even when a cumulus ensemble is not in an exactly steady state. Then from (6),

$$\frac{d}{dt} A(\lambda) \approx 0. \tag{8}$$

This constraint on $A(\lambda)$ can be used as a closure assumption as described below.

The time derivative of $A(\lambda)$ in (8) can be separated into two parts, one representing the effects of cumulus feedback on the large-scale fields and the other representing the effects of the large-scale processes. Then (8) becomes

$$\left[\frac{d}{dt}A(\lambda)\right]_{\mathrm{CU}} + \left[\frac{d}{dt}A(\lambda)\right]_{\mathrm{LS}} = \frac{d}{dt}A(\lambda), \quad \text{(9a)}$$

$$\approx 0, \quad \text{(9b)}$$

where the subscript CU refers to cumulus effects. Eq. (9b) is a statement of cumulus ensemble "cloud-work function quasi-equilibrium".

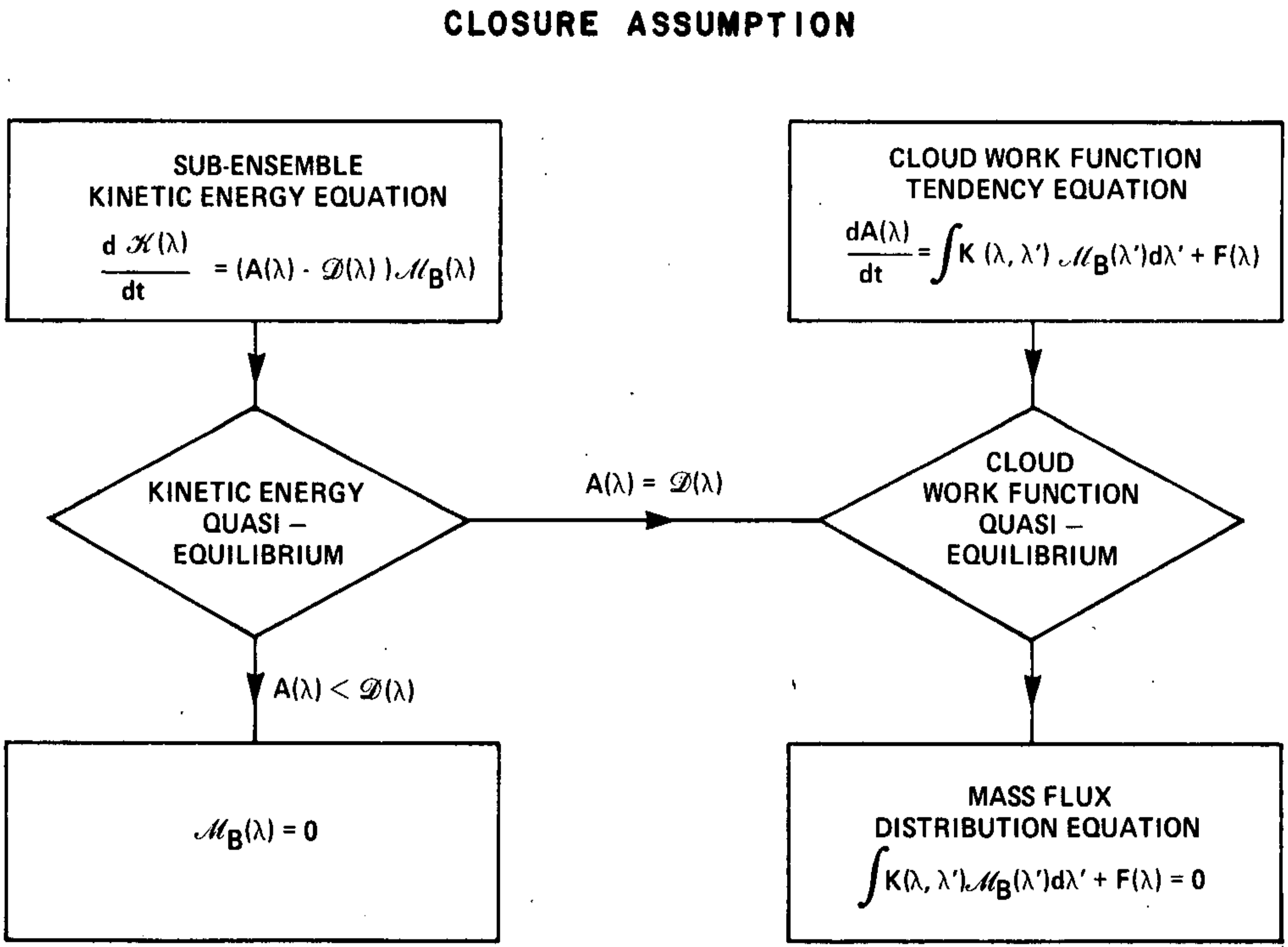
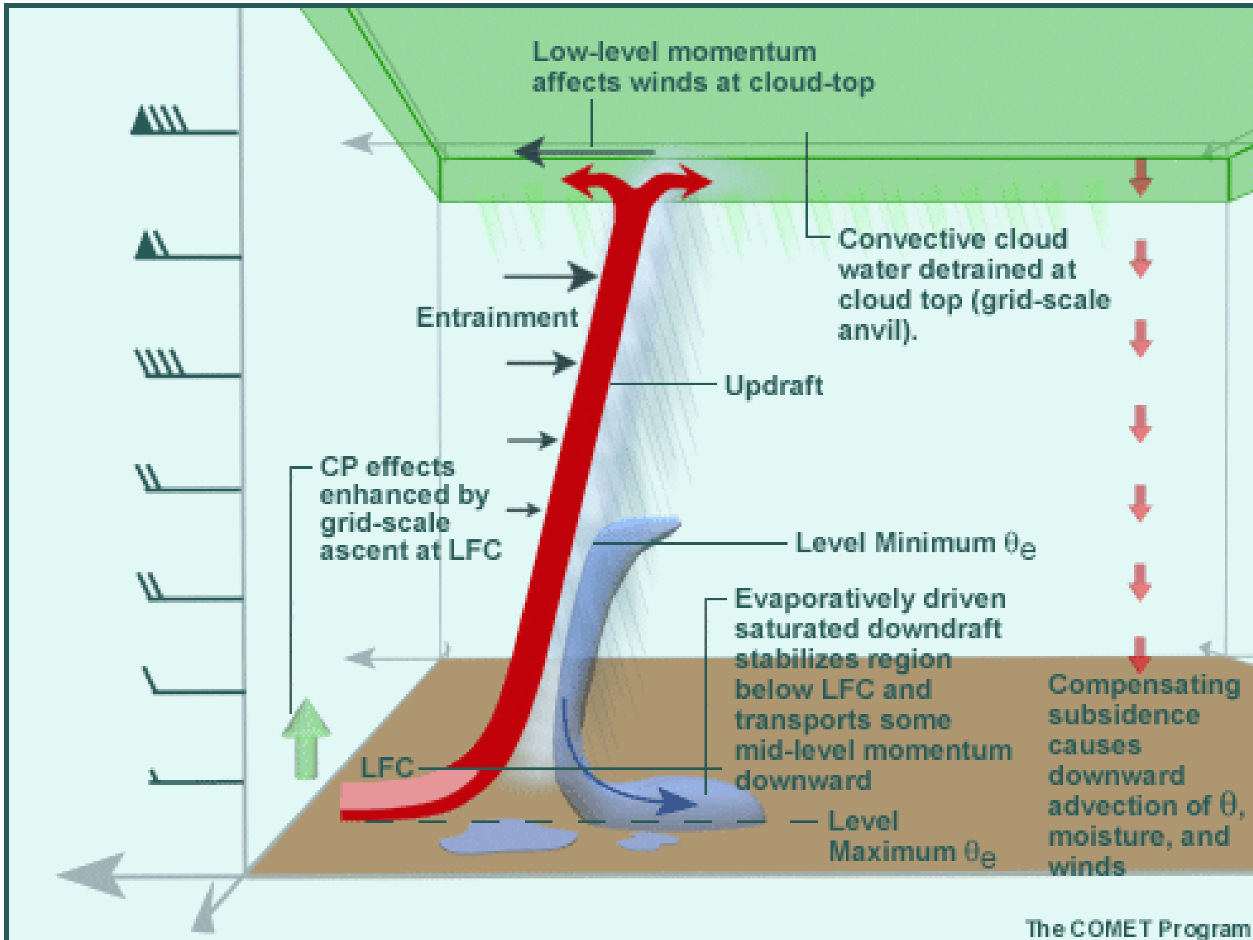


FIG. 2. A summary of the derivation of the mass flux distribution equation (10) using the kinetic energy quasi-equilibrium and the cloud-work function quasi-equilibrium.

GFS-SAS Concept .

- Similar to KF except detrains only at cloud top (remember, top is random - varies!)
- For cloud tops below equilibrium level, entrains near top to reduce buoyancy
- Momentum is also mixed: updraft source momentum detrains at top, slight downward momentum transport in compensating subsidence, more in downdraft
- Precipitation results from ascent of the source parcel. A small fraction evaporates on the way down from any level, and more evaporates into the downdraft below the θ_w minimum level



Arakawa-Schubert Scheme: Strengths & Limitations

Strengths

- Accounts for the influences of entrainment, detrainment, and compensating subsidence around clouds
- Can account for cap, depending on the specific implementation details
- Some implementations can account for saturated and/or unsaturated downdrafts, tilting of updrafts so rain falls through cloud or is ejected outside the tower, and/or microphysical processes occurring in convection
- This is a complex scheme that deals with a variety of cloud depths and is capable of providing complex sounding changes corresponding to many forecast situations

Limitations

- May not sufficiently stabilize the model atmosphere
  - May produce rain later (not immediately) or result in a prolonged period of weak convection, especially if destabilizing advection or surface fluxes counteract the modest convective scheme stabilization
  - **May result in grid-scale convection!** many serious negative forecast impacts can occur, including dramatic changes to the model's mass fields
  - It can enable high-resolution models to simulate a buoyancy-driven mesoscale circulation as exists in MCSs
- Is not designed for elevated convection
- Assumes that convection exists over only a very small fraction of the grid column, which may not be appropriate at today's higher-resolution models
- Assumes that convective updrafts entrain through the sides, whereas observations of cumulus and towering cumulus indicate entrainment mainly through cloud top. This affects scheme rainfall and heating profiles, which feed back onto the resolved motions
- Takes longer to run than other schemes

Betts-Miller-Janjic' (BMJ) Scheme: Description,& Trigger

Description: This scheme adjusts the sounding toward a pre-determined, post-convective reference profile derived from climatology.

Trigger: Three conditions are required to trigger convection:

- At least some CAPE
- Convective cloud depth exceeding a threshold value
- Moist soundings to activate

Betts, A. K., and M. J. Miller (1986), A new convective adjustment scheme. Part II: Single column tests using GATE wave, BOMEX, and arctic air-mass data sets, Q. J. R. Meteorol. Soc., 112, 693–709.

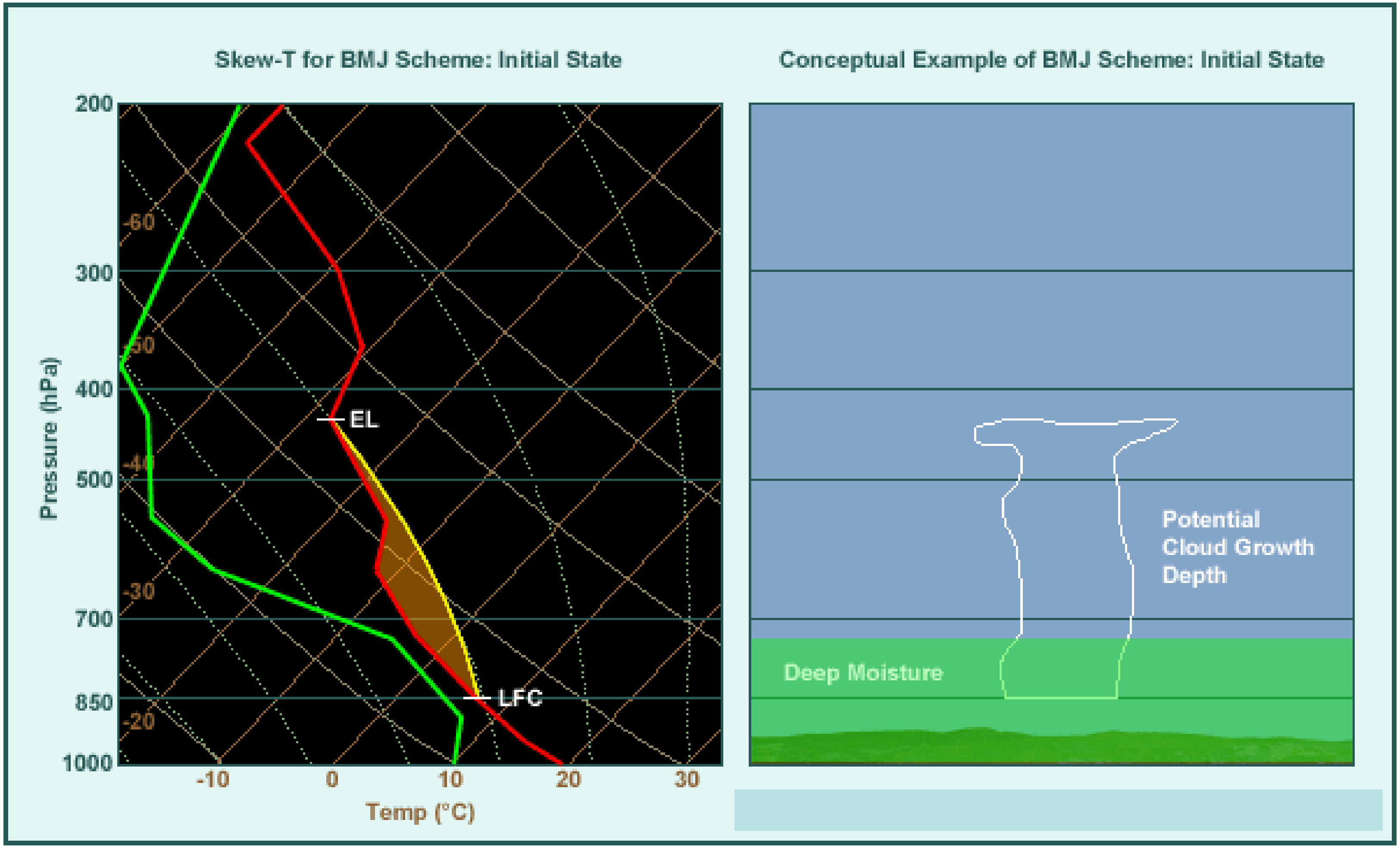
3. Betts – Miller scheme

- **Betts 1986, Betts and Miller 1986**
- **Basic idea:**
- To relax temperature and mixing ratio profile back to reference profiles in the unstable layer.
- $$\frac{\partial T}{\partial t} = \frac{T_R - T}{\tau} \qquad \frac{\partial q}{\partial t} = \frac{q_R - q}{\tau}$$ *R* represent reference profile, τ is relaxation
- time scale.
- Deep convection and shallow convection are considered separately:

→ Deep convection: if the depth of the convective layer exceeds a specified value. The reference profile are empirically determined from observations.

→ Shallow convection: when the depth of the convective layer is less than the value, it will not produce precipitation.

- **Limitations:**
- A fixed reference profile of RH may cause problems in climate models.
- Changes below cloud base have no influence.

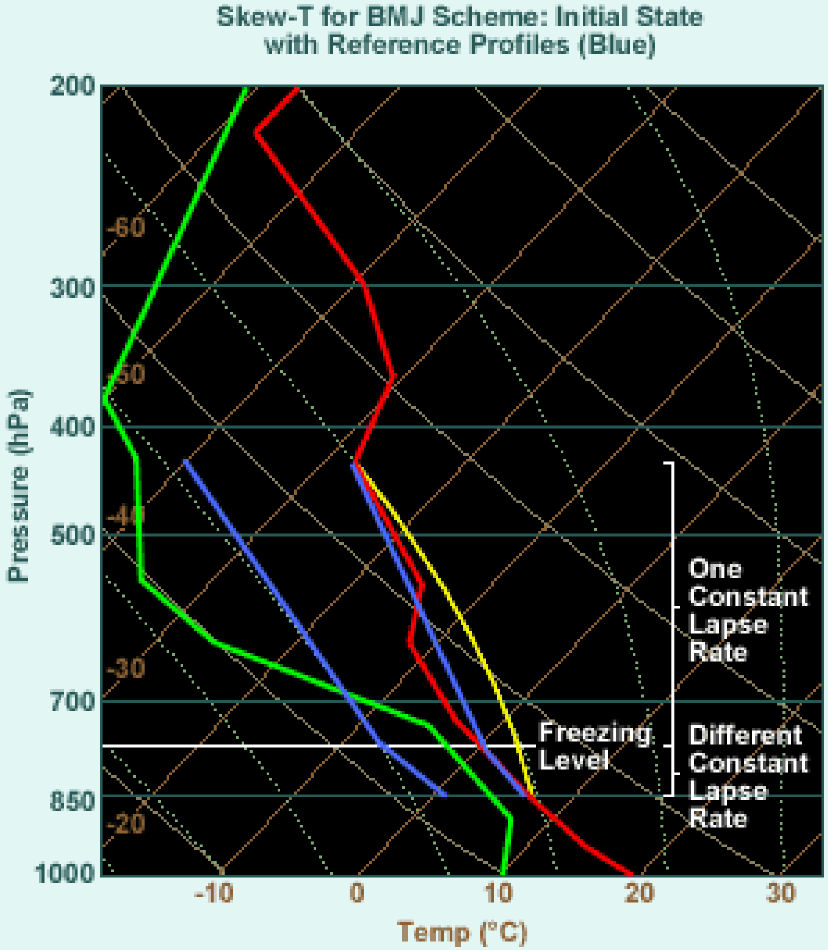


Skew-T for BMJ Scheme: Initial State
Conceptual Example of BMJ Scheme: Initial State
200
300
400
500
700
850
1000
Pressure (hPa)
-60
-50
-40
-30
-20
EL
LFC
-10
0
10
20
30
Temp (°C)
Potential
Cloud Growth
Depth
Deep Moisture

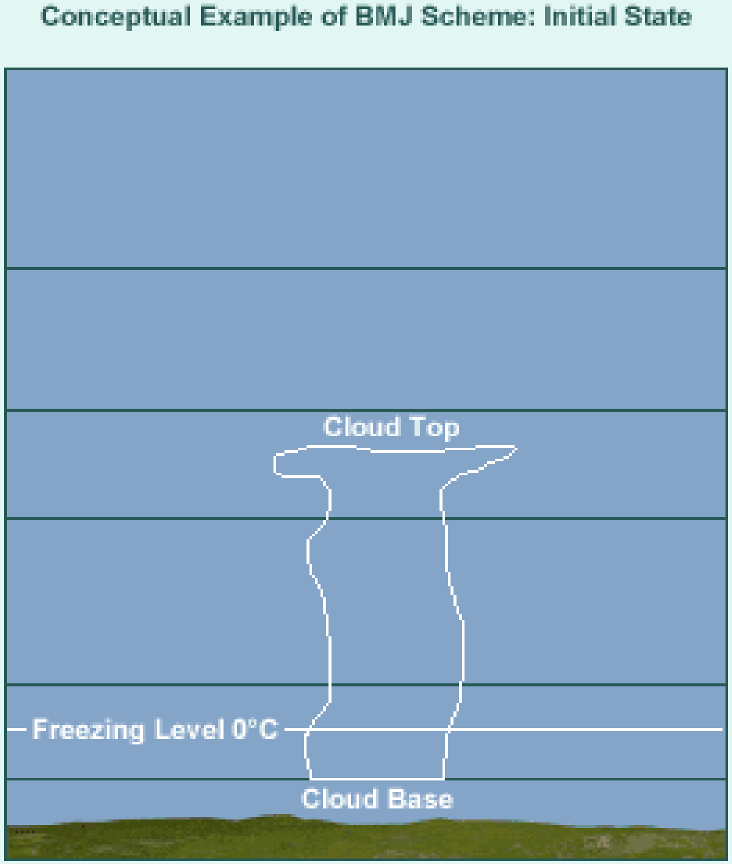
Convective changes: Starts with a reference profile, then adjusts the original sounding toward it.

A reference profile is a climatologically defined **post-**convective state, defined by points at the cloud base, cloud top, and freezing level.

Different reference profiles can be constructed and employed by the scheme as needed (for example, it can be useful to have different ones for different seasons and for the extratropics versus deep tropics).

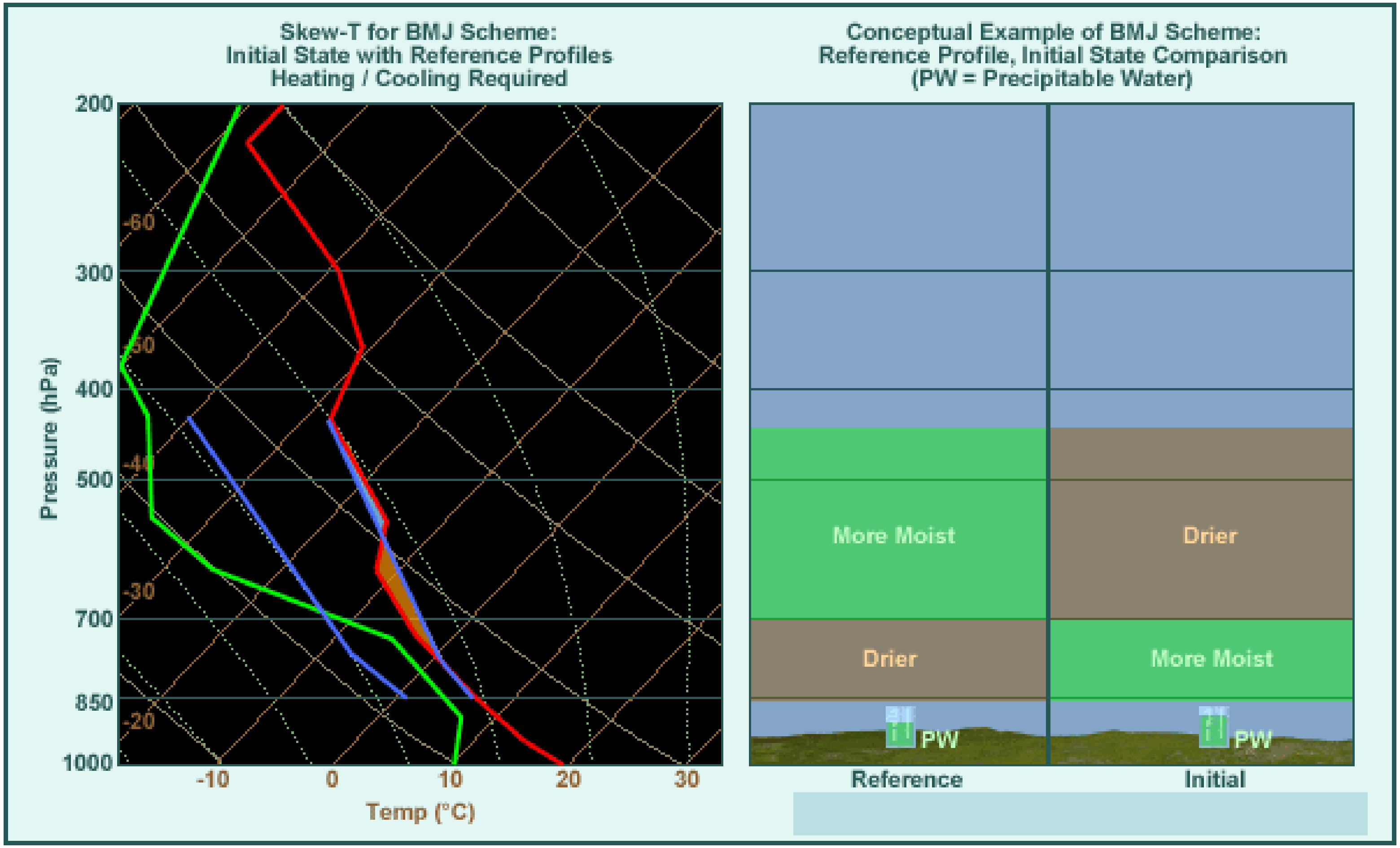


Skew-T for BMJ Scheme: Initial State
with Reference Profiles (Blue)
200
300
400
500
700
850
1000
Pressure (hPa)
-60
-50
-40
-30
-20
-10
0
10
20
30
Temp (°C)
Freezing Level
One Constant Lapse Rate
Different Constant Lapse Rate



Conceptual Example of BMJ Scheme: Initial State
Cloud Top
Freezing Level 0°C
Cloud Base

Compared to the initial sounding, the reference sounding has a different amount of precipitable water and some net heating or cooling.



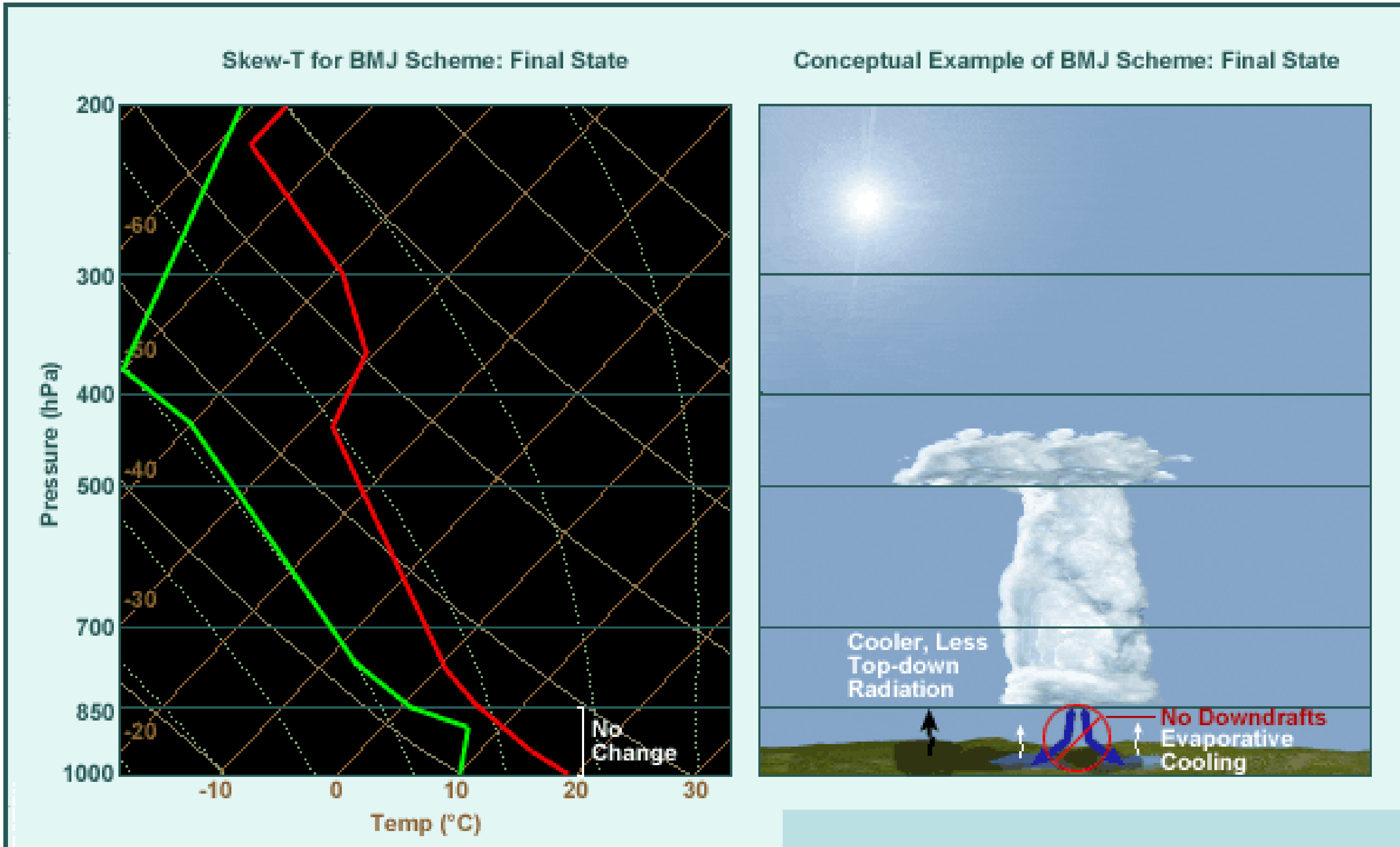
Rain is produced from a reduction in precipitable water going from the original sounding to the reference sounding.

The latent heating produced by squeezing the water out of the air must be consistent with the net warming in the temperature profile.

The reference temperature and dewpoint profiles slide in tandem left or right on the sounding until a position is found where the latent heating produced by the scheme precipitation is consistent with the sensible heating changes to the sounding.

Betts-Miller-Janjic' Scheme: Link to Large-scale Forcing & Final State

Final state: Evolves to the reference profile. Even with other model forcing, the model output soundings closely resemble the reference profiles. Note that the sub-cloud temperature and moisture profiles are not changed by the CP scheme. The scheme has no downdraft cooling; however, other non-CP processes (such as a reduction in incoming solar radiation or evaporative cooling of model precipitation) may act to cool the low levels.


Skew-T for BMJ Scheme: Final State
Conceptual Example of BMJ Scheme: Final State
Pressure (hPa)
200
300
400
500
700
850
1000
-60
-50
-40
-30
-20
-10
0
10
20
30
Temp (°C)
No Change
Cooler, Less Top-down Radiation
No Downdrafts
Evaporative Cooling

Summary

- BMJ scheme is lagged convective adjustment scheme
- The model temperature and moisture profiles are adjusted towards reference profiles which are in quasi equilibrium state.
- Three adjustment parameters are used for the construction of a reference profile. The adjustment or relaxation time period, stability weight and reference moisture profile.
- For the computation of reference moisture profile, sub-saturation level pressure values (p) are computed in terms of cloud efficiency (E). For this purpose, two sets of p profiles are defined namely p_{slow} (moist) corresponding to cloud efficiency 0.1 and p_{fast} (dry) corresponding to E=1.0
- The p values are then computed at three representative levels namely cloud base, freezing level and cloud top as follows

$$p_{b/f/t} = p_{slow(b/f/t)} + \frac{\mathrm{E} - \mathrm{E}_1}{\mathrm{E}_2 - \mathrm{E}_1}\left(p_{fast(b/f/t)} - p_{slow(b/f/t)}\right)$$

Betts-Miller-Janjic' Scheme: Strengths & Limitations

Strengths

- Often works well in moist environments with little cap
- Treats elevated convection better than other CP schemes
  Implicitly includes the effects on cloud layers of downdrafts, latent heat of fusion from freezing in updrafts, melting of falling precipitation, and many other complicating natural features
- Runs quickly; does not require much computing resources

Limitations

- Reference profiles are fixed based on climatological observations rather than being flexible for every forecast situation; as a result, they may eliminate important vertical structure
- Is only triggered for soundings with deep moisture. (This is a potential problem in arid environments)
- When triggered, the scheme often rains out too much water, either because the reference profile is too dry for the forecast situation or the transition to the reference profile is too rapid. This leaves too little water vapor behind for precipitation occurring later or downstream
- Does not account for the strength of cap-inhibiting convective development
- Does not account for any changes below cloud base
  - Makes no attempt to simulate gust fronts and their associated mesohighs
  - Only affects surface conditions indirectly, such as through the evaporation of precipitation and the reduction of solar heating from cloud cover

Kain-Fritsch Scheme

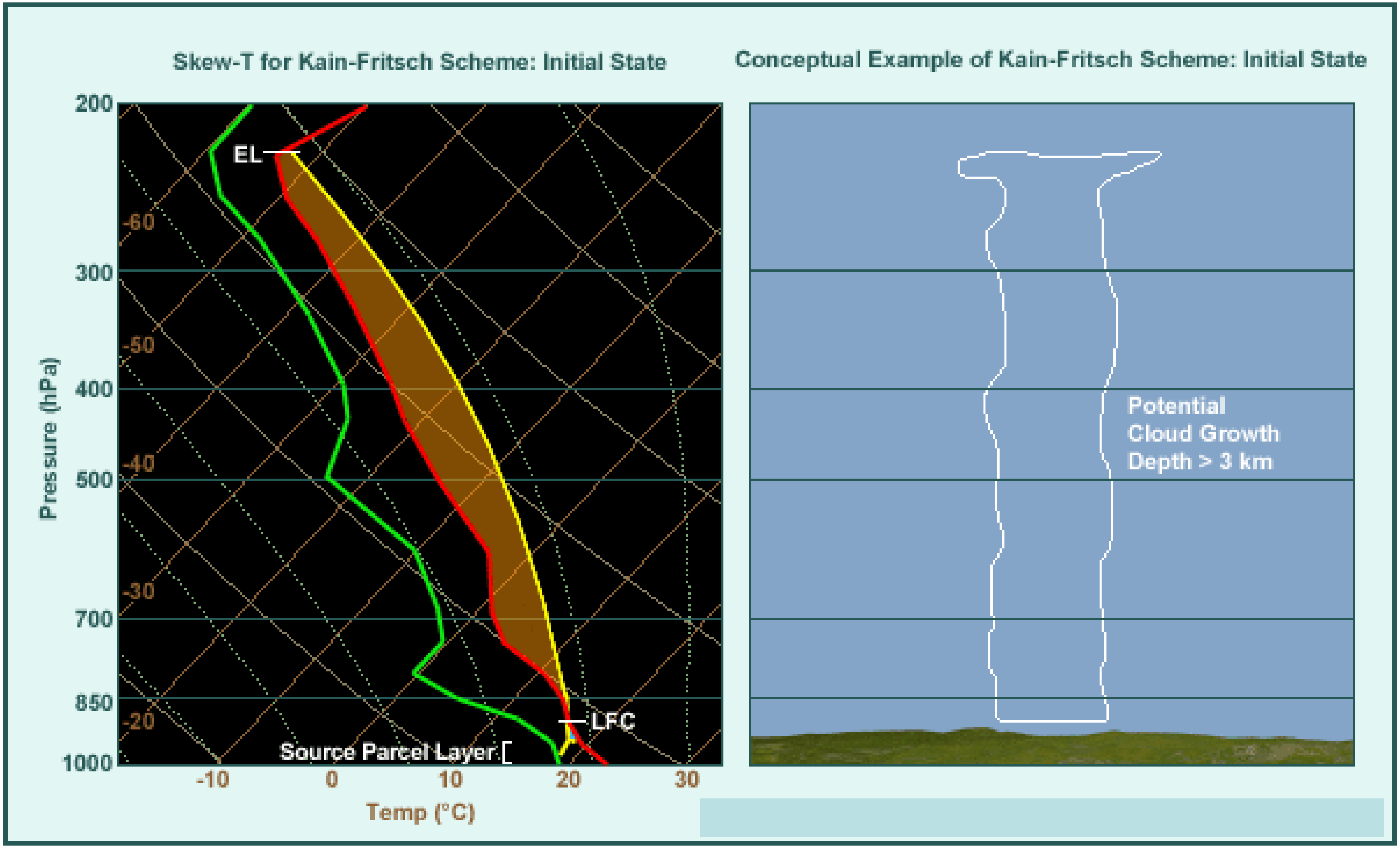
Kain, J. S., and J. M. Fritsch (1990), A one-dimensional entraining/ detraining plume model and its application in convective parameterization, J. Atmos. Sci., 47, 2784–2802,

The convection is determined by convective available potential energy (CAPE) at a grid point.

A trigger function is based on the resolvable scale vertical motion. When the scheme is activated CAPE is removed by rearrangement of temperature and moisture fields.

Trigger: The following conditions must be met for the scheme to trigger convection:

- The sounding has CAPE for source parcels from a low-level layer 50 to 100 hPa thick
- The cap is small enough for a parcel to penetrate given a boost of a few m/s (a function of large-scale vertical motion at LCL)
- The convective cloud depth exceeds a threshold



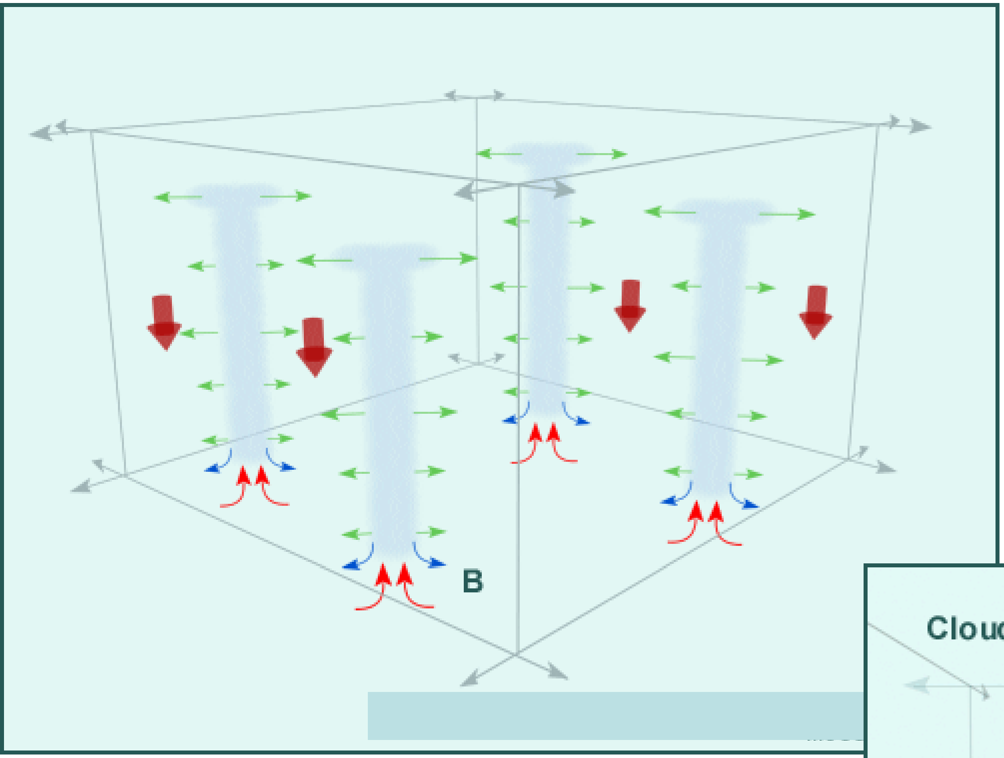
Skew-T for Kain-Fritsch Scheme: Initial State
Conceptual Example of Kain-Fritsch Scheme: Initial State
Pressure (hPa)
200
300
400
500
700
850
1000
-60
-50
-40
-30
-20
EL
LFC
Source Parcel Layer
-10
0
10
20
30
Temp (°C)
Potential
Cloud Growth
Depth > 3 km

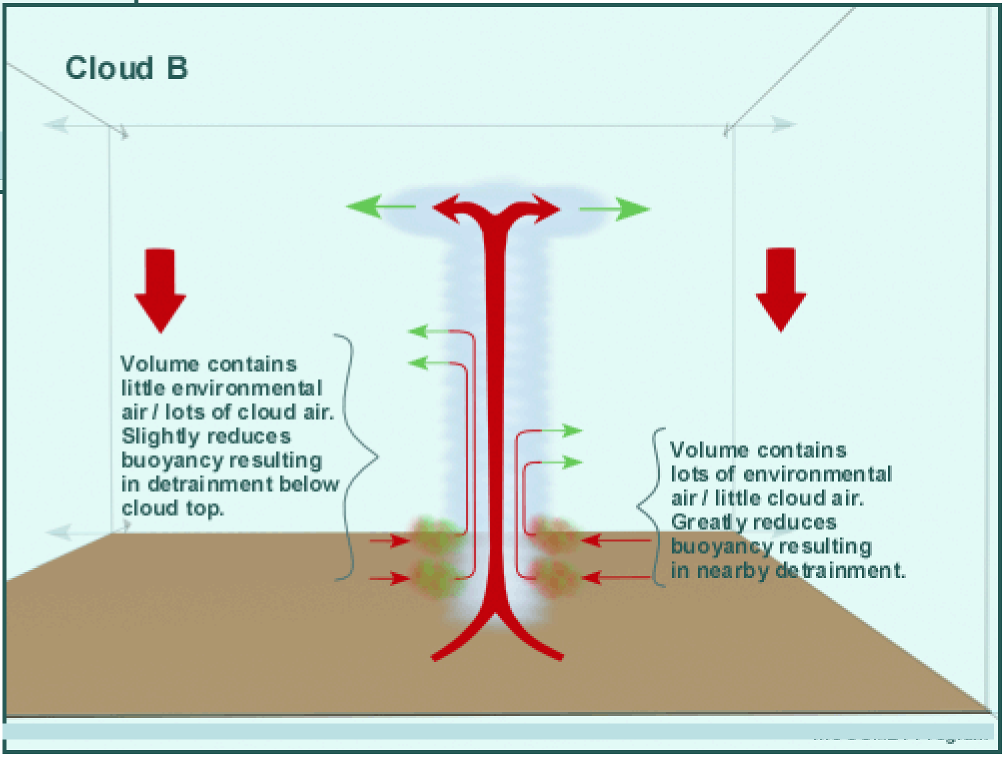
Kain-Fritsch Scheme: Convective Changes

Like the Arakawa-Schubert scheme, changes to the sounding result from cloud detrainment, environmental subsidence, and evaporatively driven downdrafts dumping into the convecting source layer. In addition, like AS, these effects are based on cloud properties determined in a one-dimensional cloud model

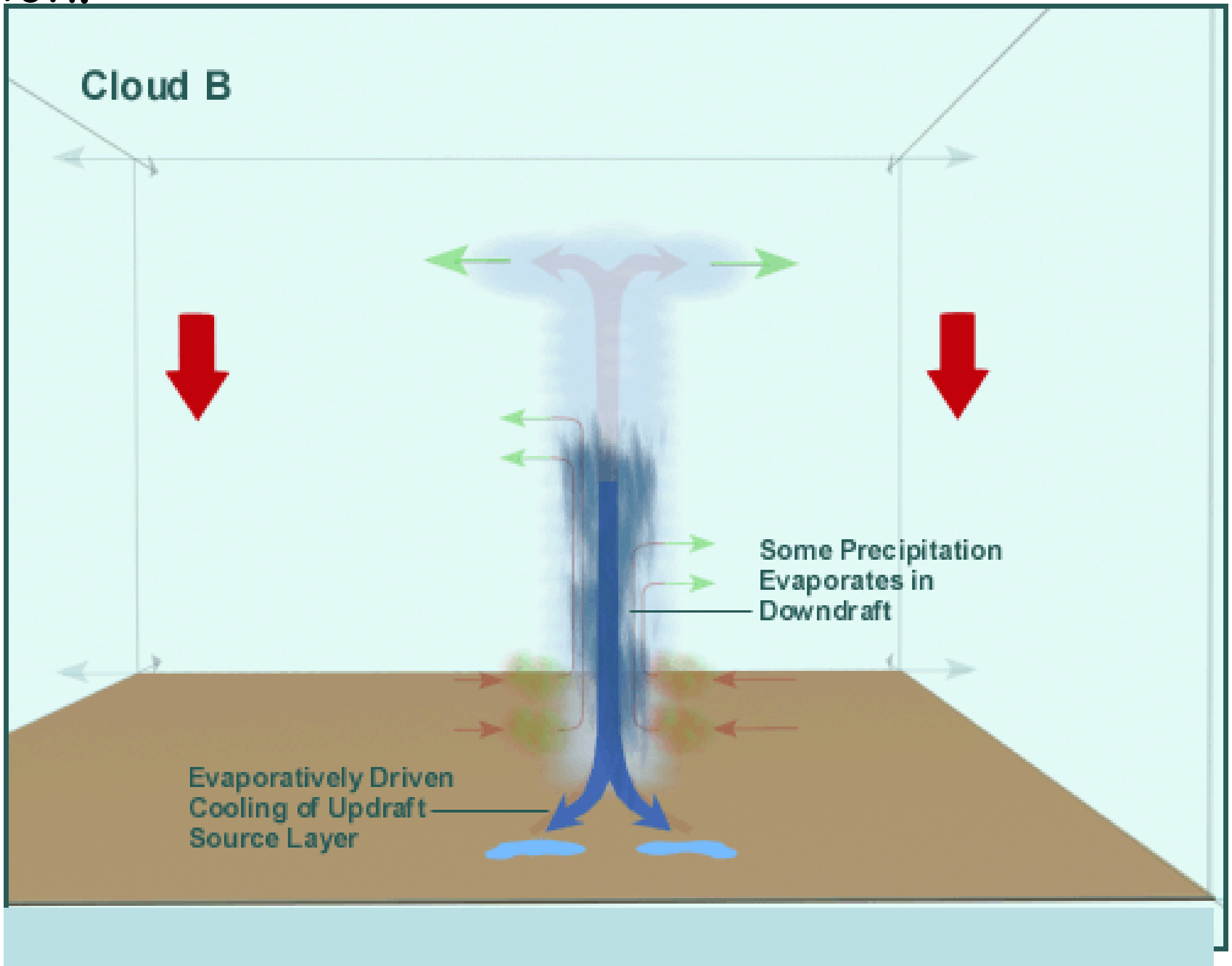
Unlike the Arakawa-Schubert scheme, clouds of only one height (the tallest cloud that the sounding permits) are assumed to exist and entrain and detrain at many levels. Instead of a single mixture of cloud and environment, entrainment is assumed to produce many different mixtures, which have different buoyancy properties and thus detrain at different levels. This allows the scheme to be even more responsive and sensitive to different soundings than AS.

As per Bechtold et al. [2001, equation (5), p. 873], the instability of the moist air parcel for deep convection is triggered/suppressed by a temperature perturbation (DT), which is a function of grid-scale motion and defined by DT = ± $c_w|w_n|1/3$ with c_w = 6 K $m^{-1/3}s^{1/3}$, where $w_n=A^{1/2}/Dx_{ref}w$ is the normalized large-scale vertical velocity using a reference grid space of Dx_{ref} = 25 km.


B

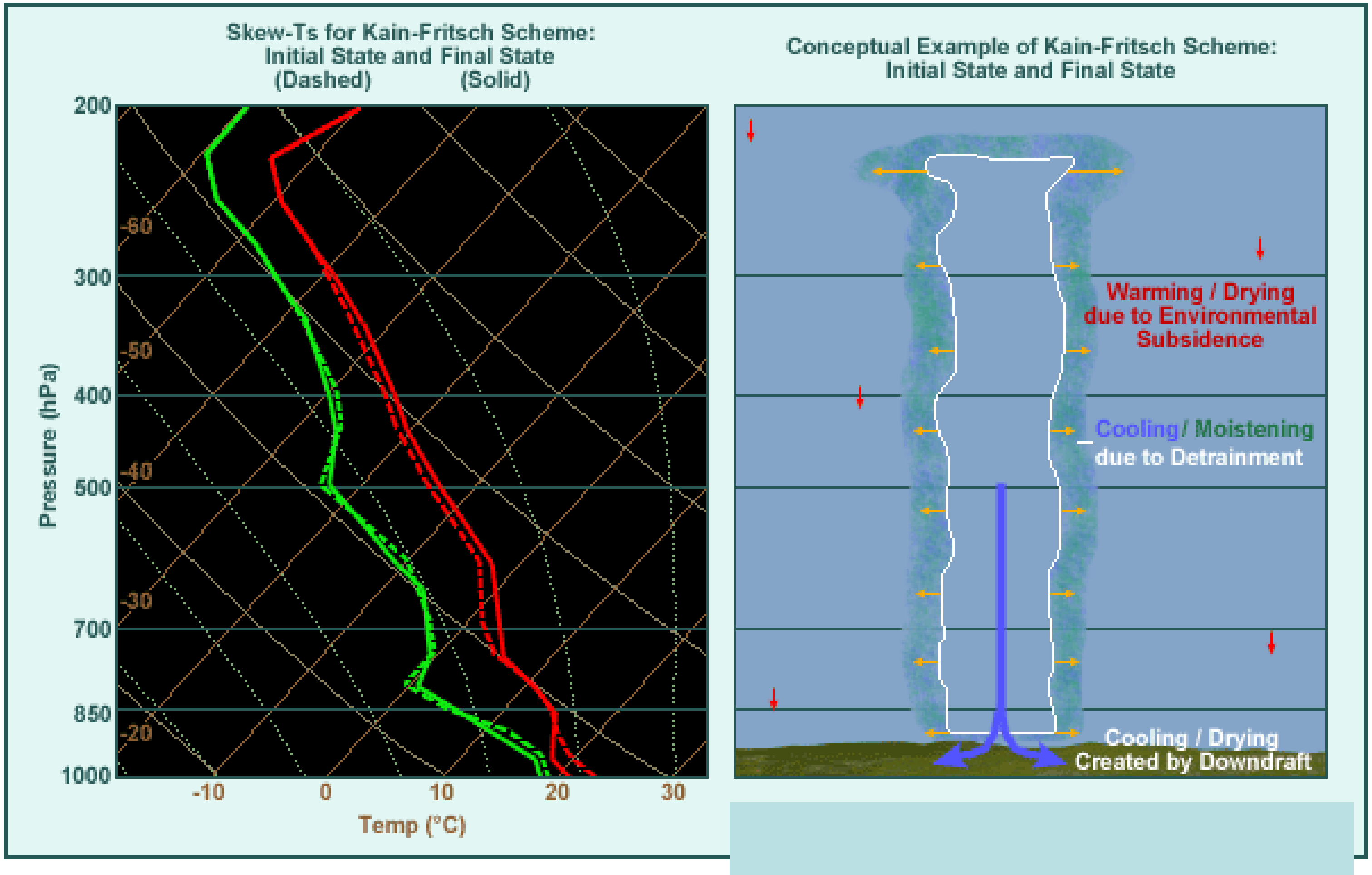

Cloud B
Volume contains little environmental air / lots of cloud air. Slightly reduces buoyancy resulting in detrainment below cloud top.
Volume contains lots of environmental air / little cloud air. Greatly reduces buoyancy resulting in nearby detrainment.

Like in the Arakawa-Schubert scheme, precipitation is produced in the cloud model, with some precipitation evaporating in the downdraft and some instantly falling as precipitation.



The two primary differences between AS and KF are in the triggering process (determining where and when convection forms) and the link to the large scale (determining the intensity of the changes). Both have the same mass-flux approach of accounting for the fundamental grid-scale effects of convection (cloud detrainment, downdrafts, and environmental subsidence). In addition, both are highly sensitive to modeler-selected parameters in the cloud models that are used to calculate these effects.

Sounding changes are the sum of the effects of compensating subsidence, cloud sources at detrainment levels, and downdrafts. These are applied at a constant rate (taking no account of environment changes) over a pre-specified time period that represents a convective cell life cycle.



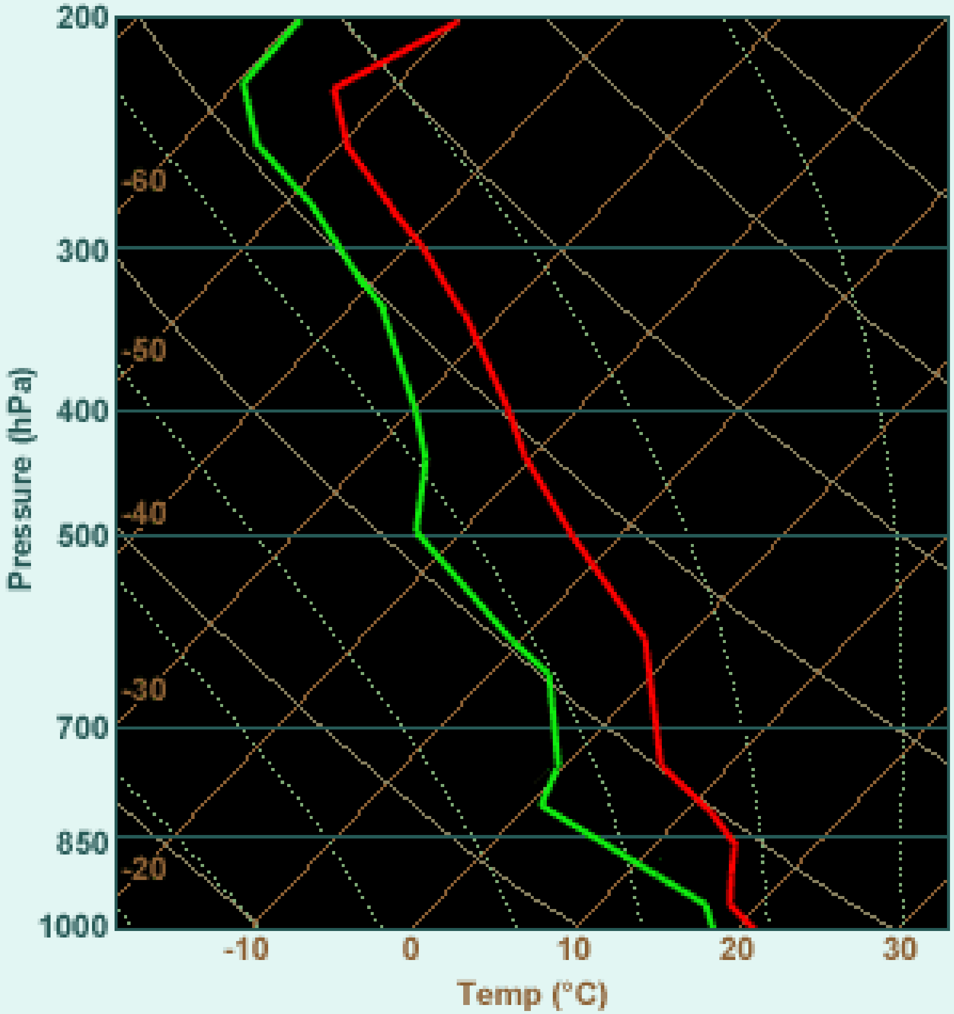
Kain-Fritsch Scheme: Link to Large-scale Forcing & Final State

Link to large scale forcing: Large-scale vertical velocity at the LCL contributes to determining where convection is triggered. Once activated, the scheme entirely consumes CAPE in the 50- to 100-hPa thick triggering source layer during a 30- to 60-minute convective cycle. The CAPE in other layers may be used in triggering another round of convection after this cycle ends,

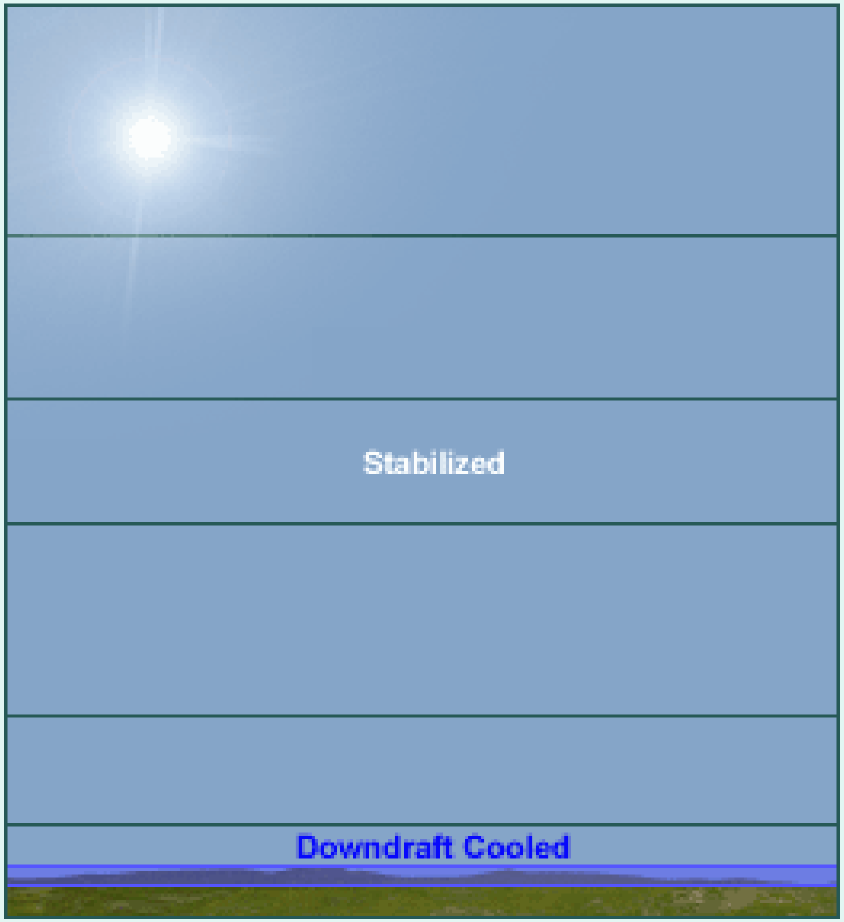
Final state:

Sounding changes occur after source-layer CAPE has been depleted during the 30- to 60-minute convective cycle.

Skew-T for Kain-Fritsch Scheme: Final State



Conceptual Example of Kain-Fritsch Scheme: Final State



Kain-Fritsch Scheme: Strengths & Limitations

Strengths

- Suitable for mesoscale models and.
- The assumption about consuming CAPE is appropriate for short time and space scales
- It accounts for microphysical processes in convection; can be set up to feed hydrometeors to the PCP scheme
- May perform better in cases of severe convection
- Physically realistic in many ways
- Has the most realistic treatment of trigger and cap
- Accounts for entrainment and detrainment more realistically than Arakawa-Schubert schemes
- Like the Arakawa-Schubert scheme, can vary its response to different forecast scenarios

Limitations

- Tends to leave unrealistically deep saturated layers in post-convective soundings

- Takes longer to run than simpler schemes

- The assumption about the rapid consumption of CAPE is not appropriate for coarse-resolution models, such as climate models

- Convection triggers in scattered grid boxes. (Other schemes tend to have a smoother clustering of grid boxes where convection is triggered.) Although this may be more realistic, it can make the interpretation of model fields more difficult

Convective Parameterization Impacts

- Model convective precipitation is only created as a by-product of the CP scheme rearranging heat and moisture, yet it affects the model's precipitation forecast and the model's soil moisture availability, which can then affect evaporation and subsequent boundary-layer dewpoints and CAPE
- Incorrect timing, placement, and amount of model precipitation can cause errors in the simulation of many forecast variables, especially if they are treated in a consistent, physically realistic manner
- Unlike actual convection, most CP schemes do not change the winds and none directly affect the vertical motion. Winds, however, can change in response to the heating created by the latent heat released when a scheme is active. The heating and moisture changes induced by CP schemes result in changes to the height field and, in turn, the winds.
-

- The primary purpose of a CP scheme is to reduce instability so the model does not produce excessive grid-scale precipitation and all of the associated adverse forecast impacts.

- Precipitation is produced as a necessary by-product of the CP scheme removing instability.

References

- Arakawa, A., and W.H. Schubert, 1974: Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. *J. Atmos. Sci.*, **31**, 674-701.
- Betts, A. K., 1986: A new convective adjustment scheme. Part I: Observational and theoretical basis. *Quart. J. Roy. Meteor. Soc.*, **112**, 677-691.
- Betts, A. K., and M. J. Miller, 1986: A new convective adjustment scheme. Part II: Single column tests using GATE wave, BOMEX, ATEX, and Arctic air-mass data sets. *Quart. J. Roy. Meteor. Soc.*, **112**, 693-709.
- Janjic', Z.I., 1994: The step-mountain eta coordinate model: Further developments of the convection, viscous sublayer, and turbulence closure schemes. *Mon. Wea. Rev.*, **122**, 927-945.
- Kain, J. S., and J.M. Fritsch, 1992: The role of convective "trigger function" in numerical forecasts of mesoscale convective systems. *Meteorol. Atmos. Phys.*, **49**, 93-106.
- Kain, J. S., and J.M. Fritsch, 1990: A one-dimensional entraining/detraining plume model and its application in convective parameterization. *J. Atmos. Sci.*, **47**, 2784-2802.
- Kuo, H. L., 1974: Further studies of the parameterization of the influence of cumulus convection on large-scale flow. *J. Atmos. Sci.*, **31**, 1232-1240.

Which of the following statements about Convective parameterization schemes are true?

a) The primary function is to remove excess instability

b) The primary function is to forecast convective precipitation

c) They produce precipitation as a "by-product"

d) Most schemes do not directly modify the horizontal wind field

e) They do not directly affect the vertical motion field

What is the principle of formulation of KUO parameterization scheme ?

What is “bull’s eye” in relation to KUO scheme?
What is the trigger of KUO scheme?

What is the fundamental difference of KUO scheme and Arakawa-Schubert scheme?

What is the physical meaning of “Cloud work function”?

Why the AS scheme is also known as semi prognostic scheme?

What is the constraint of kinetic energy budget for each sub cloud ensemble?

How does AS scheme couple the cloud scale and large scale?

Can a cloud grow when the cloud scale kinetic energy by large scale buoyancy force is less than the cloud scale dissipation and when cloud base mass flux is zero? Explain

Mention some strengths and limitation of AS scheme

- What is the convective trigger in BMJ?
- Does this scheme account for sub cloud layer processes? Expalin
- Mention its merits and demerits?
- What is the principle on which the KF scheme is formulated
- What is the likelihood of KF scheme's performance during monsoon regime